

**Woods Hole  
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**Monthly Maps of Sea Surface Height in the North Atlantic  
and Zonal Indices for the Gulf Stream Using  
TOPEX/Poseidon Altimeter Data**

by

Sandipa Singh and Kathryn A. Kelly

June 1997

**Technical Report**

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Monthly Maps of Sea Surface  
Height in the North Atlantic and  
Zonal Indices for the Gulf Stream  
Using TOPEX/Poseidon Altimeter  
Data

Sandipa Singh  
Kathryn A. Kelly

Woods Hole Oceanographic Institution  
Woods Hole, MA

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## Abstract

Monthly maps of sea surface height are constructed for the North Atlantic ocean using *TOPEX/Poseidon* altimeter data. Mean sea surface height is reconstructed using a weighted combination of historical, hydrographic data and a synthetic mean obtained by fitting a Gaussian model of the Gulf Stream jet to altimeter data. The resultant mean shows increased resolution over the hydrographic mean, and incorporates recirculation information that is absent in the synthetic mean. Monthly maps, obtained by adding the mean field to altimeter sea surface height residuals, are used to derive a set of zonal indices that describe the annual cycle of meandering as well as position and strength of the Gulf Stream.

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## 2 RECONSTRUCTING THE MEAN SEA SURFACE HEIGHT

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### 1 Introduction

This report describes the construction of monthly maps of total sea surface height (*ssh*) in the North Atlantic ocean using *TOPEX/Poseidon* altimeter data. These maps are then used to derive zonal indices that describe the transport, position, velocity, and curvature of the Gulf Stream.

Collinear analysis of the subtrack data is described extensively in [1]. The end-product of this analysis is cleaned residual *ssh* profiles splined along-track on to a common latitude grid. Since the mean is removed along with the geoid in the collinear analysis, it is first necessary to recover the mean height field in order to measure total *ssh* difference. The first section of this report deals with the reconstruction of a mean for the North Atlantic using both historical hydrographic data and a synthetic mean derived using the Kelly/Gille/Qiu method [4], [5]. This mean is used along with the *ssh* residuals to obtain monthly maps of *ssh*. The second section describes the calculation of zonal indices that characterize the state of the Gulf Stream (GS) from these monthly maps. Contour plots of the monthly maps are included in the Appendix.

### 2 Reconstructing the Mean Sea Surface Height

For the purposes of this study, the subtracks that fell between latitudes 20°N to 60°N and longitudes 30°W to 80°W were analyzed for cycles 4 through 80 which spanned a time period from October 1992 to November 1994. Cycles one through three were discarded because of poor data quality. Figures 1 and 2 show the coverage of ascending and descending ground tracks in this area along with the *ssh* variance along the tracks. The peak of the variance reveals the position of the GS jet.

Two separate calculations of the mean are described in the following subsections.

#### 2.1 Obtaining the hydrographic mean

To obtain a climatological mean, we used the Lozier/Owens/Curry HydroBase data set which calculates the mean dynamic height by gridding raw, historical data for pressure, temperature and salinity on density surfaces [3]. This yields netCDF files which are then used to extract the dynamic height of the surface relative to a reference depth of 2000 meters. Figure 3 shows a contour plot of these data.

#### 2.2 Obtaining a synthetic mean

To generate a synthetic mean using the Kelly/Gille/Qiu method, the Gulf Stream velocity was modeled as a Gaussian jet [4]. Assuming that the GS meanders a distance

## 2 RECONSTRUCTING THE MEAN SEA SURFACE HEIGHT

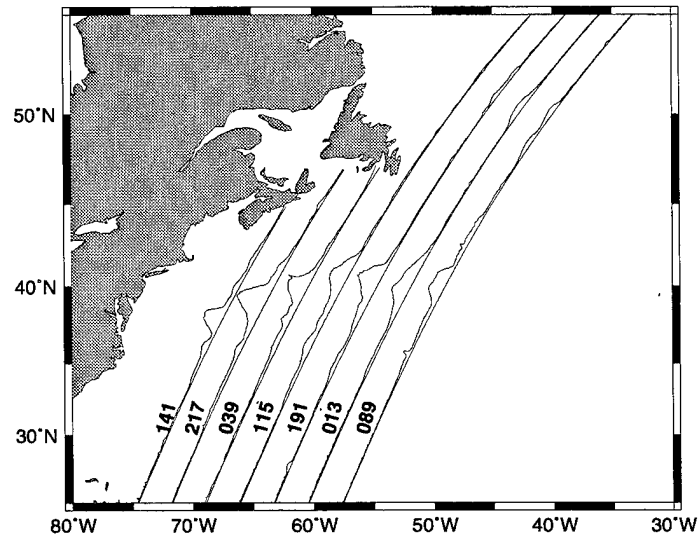


Figure 1: *TOPEX/Poseidon* ascending ground track coverage for the North Atlantic showing *ssh* variance along track.

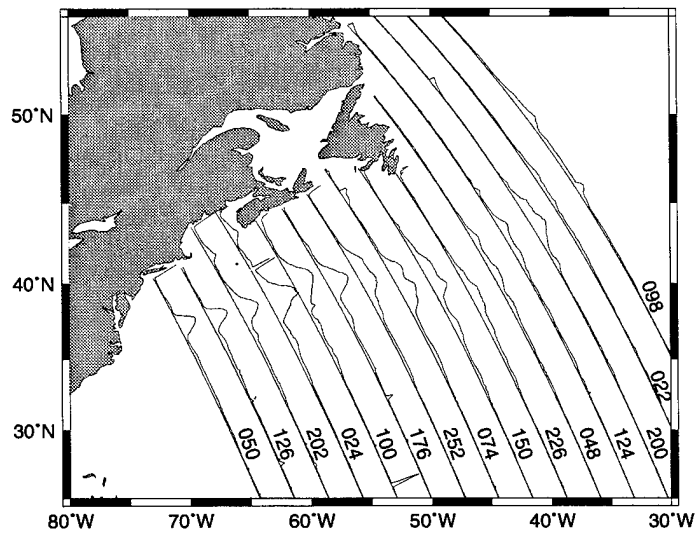


Figure 2: *TOPEX/Poseidon* descending ground track coverage for the North Atlantic showing *ssh* variance along track.

## 2 RECONSTRUCTING THE MEAN SEA SURFACE HEIGHT

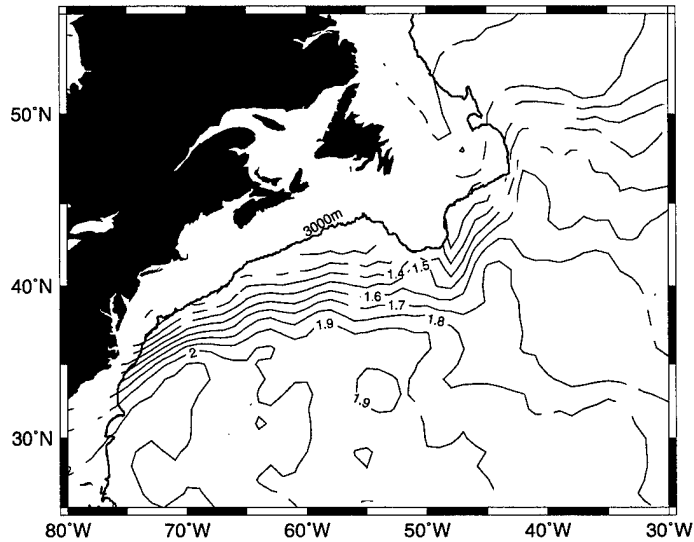


Figure 3: Dynamic height from historical hydrographic data, reference level 2000 meters.

at least as large as its width, the instantaneous *ssh* residuals will contain information on the magnitude and position of the jet. These, combined with an initial guess of the mean strength, define the Gaussian velocity jet completely. A least squares fit to the Gaussian jet was performed on the *ssh* residuals for each subtrack and the parameters of the fit modified until a convergence criterion was satisfied. This method was applied to all tracks independently and yielded a mean *ssh* field (Figure 4) that was consistent in position, but with a narrower and stronger GS than that obtained from hydrographic data.

### 2.3 Combining the hydrographic and the synthetic means

The Kelly/Gille/Qiu synthetic method only models the jet and cannot be used to recover the mean far from the GS itself. In other words, there is no information about the recirculation gyres that can be seen in the HydroBase mean. To incorporate information away from the GS while retaining the narrower width of the jet, the simple Gaussian jet model was modified. If we assume that the difference in height between the synthetic and the HydroBase mean is due to the presence of northern and southern recirculation gyres in the latter, then the recirculation gyres can be modelled as a wide, slow Gaussian jet, whose height equals this difference (Figure 5). For this model, first mean height profiles of hydrographic data were obtained by splining gridded HydroBase data along satellite subtracks. An error function fit was performed on the profiles which yielded the function,



## 2 RECONSTRUCTING THE MEAN SEA SURFACE HEIGHT

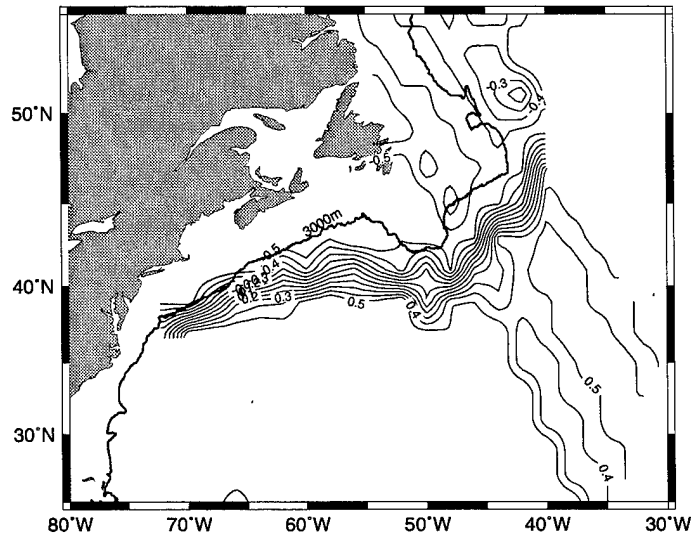


Figure 4: Synthetic *ssh* mean obtained from *TOPEX/Poseidon* data.

$erf(a_1, a_2, a_3)$ , where  $a_1, a_2, a_3$  are the amplitude, position and width parameters of the fit. The recirculation was then set to an error function  $erf(a_1 - a_1^s, a_2, 2a_3)$ , where  $a_1^s$  is the amplitude parameter of the synthetic mean height profile. This recirculation error function was subtracted from the synthetic mean profiles to yield modified synthetic profiles.

Finally, the modified synthetic profiles were gridded to  $1^\circ \times 1^\circ$  and combined with the HydroBase mean using a spatially varying function. The profiles were first interpolated between tracks to retain the narrow jet structure of the Gulf Stream and then splined onto the  $1^\circ \times 1^\circ$  grid using a biharmonic spline. The interpolation could be carried out since the mean height did not change much from track to track (except in the bifurcation region around  $50^\circ W$ ). Figure 6 shows the spacing of the original tracks as thick solid lines, the interpolated ones are thin lines, and the open circles are the  $1^\circ \times 1^\circ$  grid points.

The weighting function used for combining the HydroBase mean with the gridded mean was calculated by low-pass filtering the magnitude of the gradient of *ssh* of the synthetic mean (Figure 7). This procedure weighted the synthetic mean more heavily in regions of large velocities where the Gaussian model is expected to be more accurate, and the hydrographic mean more heavily away from the Gulf Stream, where the Gaussian model is not accurate. East of  $40^\circ W$ , where there was no synthetic mean data, the HydroBase mean was used exclusively, whereas inside the 3000 meter isobath, where no HydroBase data are available, the synthetic mean was used. The resulting map is shown

## 2 RECONSTRUCTING THE MEAN SEA SURFACE HEIGHT

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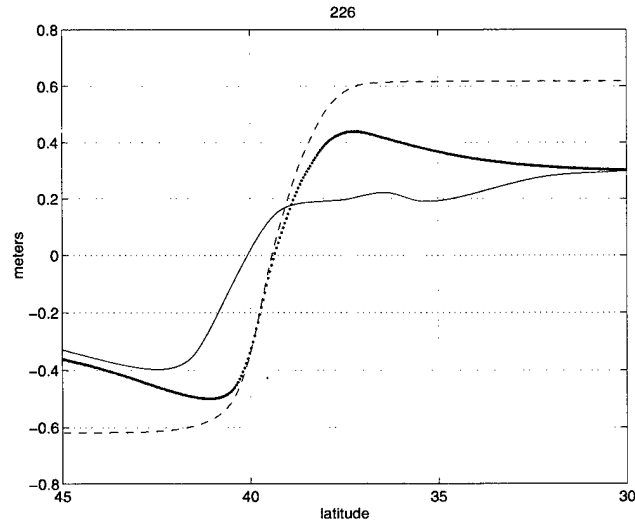


Figure 5: A comparison of total height (dashed line), HydroBase height (light line) and net height (heavy line) along subtrack 226.

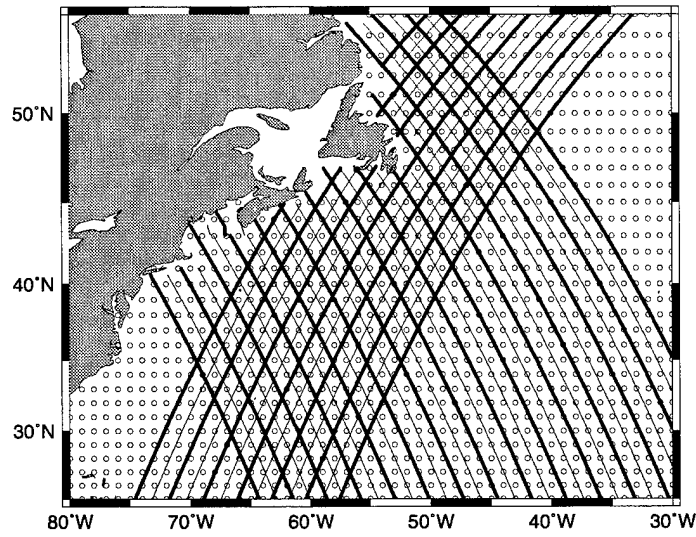


Figure 6: Heavy subtrack lines mark the initial sampling pattern of the mean height, light lines show the interpolated subtracks and circles indicate the final grid onto which it was mapped.

## 2 RECONSTRUCTING THE MEAN SEA SURFACE HEIGHT

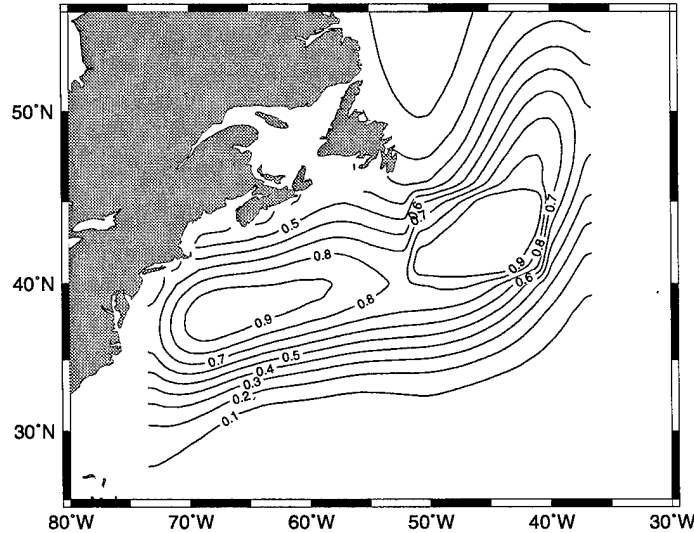


Figure 7: Weights used on the synthetic mean.

in Figure 8. The combined mean field shows increased resolution over the hydrographic mean, and incorporates recirculation information which could not previously be obtained from the synthetic method.

### 2.4 Obtaining Monthly Sea Surface Height Maps

Mean *ssh* field reconstructed in the previous step can now be added back to the *ssh* residuals to obtain monthly maps of the Gulf Stream and the North Atlantic Current. For this, the residuals need to be regridded in both time and space. Assuming that the two are independent, the two-dimensional regridding can be broken up into two separate one-dimensional problems. The residuals are first averaged in time using a boxcar filter with a width of sixty days. This puts all cycles on a monthly spacing. The monthly cycles are then splined on to a  $1^\circ$  by  $1^\circ$  grid using a two-dimensional biharmonic spline. They are then added to the means. See the Appendix for contour plots of the monthly maps so obtained.

Monthly maps derived from the earlier *Geosat* mission were also included in this study and covered a period from November 1986 to April 1989. *Geosat* subtrack data have an across-track spacing approximately half that of the *TOPEX/Poseidon* data, with a longer sampling period of 17 days. It was processed similarly but had an orbit correction applied to it [2]. Because of smaller across-track spacing, it was gridded on to a  $0.5^\circ \times 0.5^\circ$  grid. Mean sea surface height was constructed by combining the synthetic

## 2 RECONSTRUCTING THE MEAN SEA SURFACE HEIGHT

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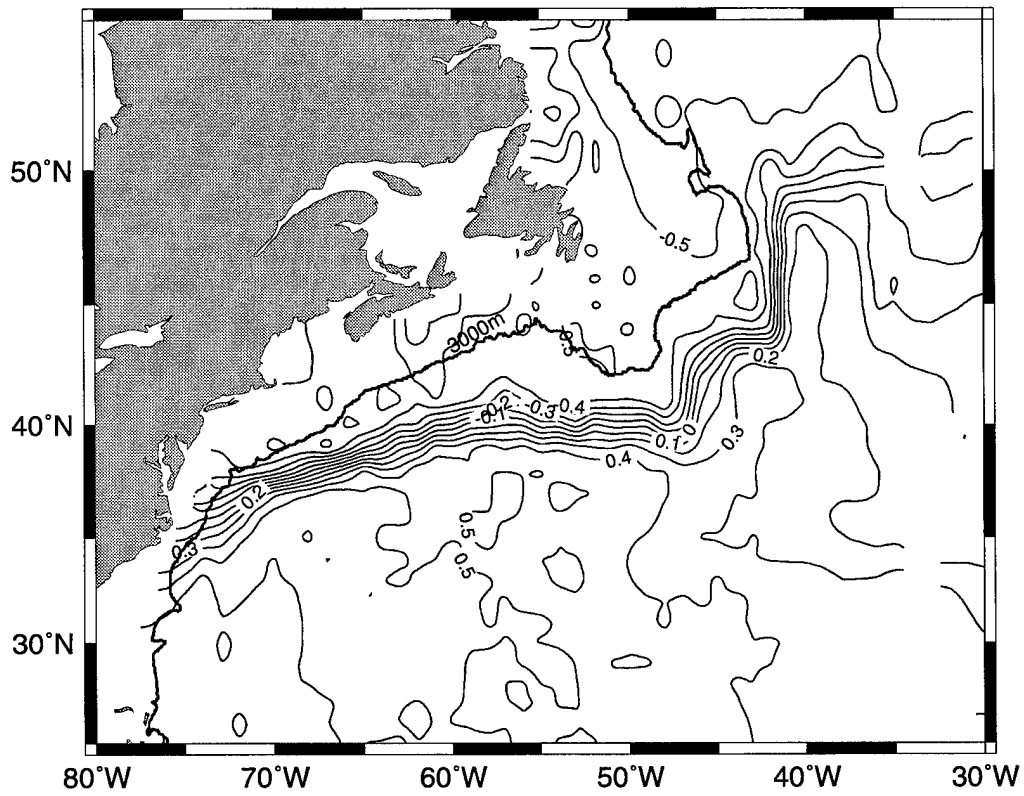


Figure 8: Weighted combination of HydroBase and synthetic means.

### 3 CALCULATING GULF STREAM INDICES

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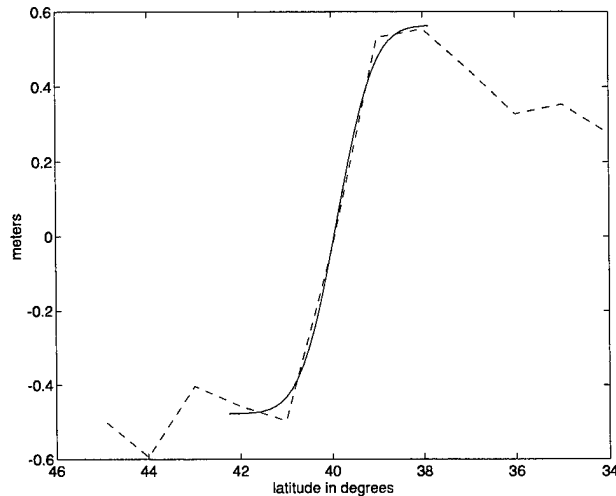


Figure 9: Error function fit to height along 50°W for November 1994.

mean with historical hydrographic data [6].

### 3 Calculating Gulf Stream Indices

To characterize the annual cycle of meandering, as well as the strength and position of the Gulf Stream, a series of zonal indices were developed from monthly maps of *ssh*. These indices covered a region from 73°W to 50°W which was further broken down into two subregions: upstream and downstream of 63°W, the approximate location of the New England Seamounts. The calculation of each index is enumerated below:

1. Transport: Height profiles along each degree of longitude are interpolated onto a 0.125 degree grid, and error functions are then fitted to them to estimate height difference across the jet. Figure 9 shows a fit for one profile. These height differences are averaged over all longitudes to get the height difference index for the month.
2. Position: If the path of the Gulf Stream is described by the zero contour of the height field, the mean position,  $P$ , is just a zonal average of the latitude of this zero contour. To get a robust estimate of path, the -0.1, 0 and 0.1 meter contours are averaged (Figure 10) before calculating  $P$ .

### 3 CALCULATING GULF STREAM INDICES

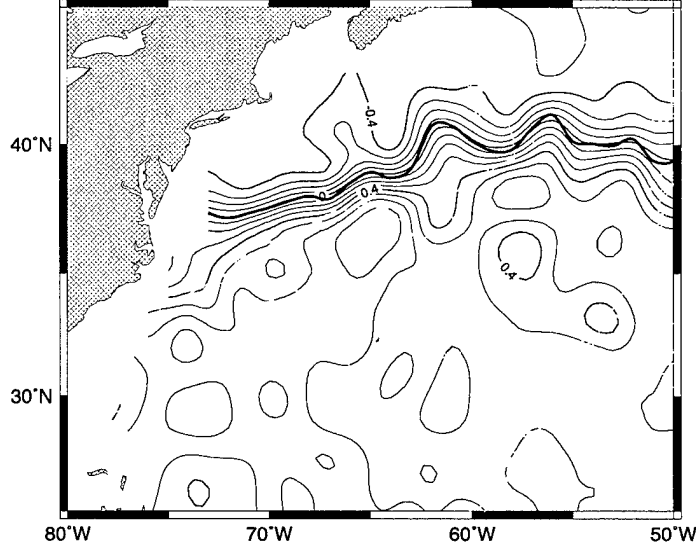


Figure 10: Path of the Gulf Stream jet (heavy black line) for November 1994, overlaid on *ssh* contours for that month. Slight differences in the path from the height contours are due to the different plotting packages used to create them.

3. Curvature: Curvature of a function  $y = f(x)$  is given by:

$$C = \frac{y''}{[1 + (y')^2]^{3/2}} \quad (1)$$

where the primes denote differentiation with respect to  $x$ . In this case,  $f$  is the path of the Gulf Stream as described in the calculation of  $P$ ,  $x$  is the longitude of the path while  $y$  is the latitude. Curvature is calculated at every point along the path, and is then averaged across the path to yield the curvature index,  $C$ .

4. Eastward Velocity: Eastward velocity is calculated along each degree of longitude. Since peak velocity estimates may be noisy, the velocity at a fixed width of one degree is used as the eastward velocity of the jet. This is averaged for all longitudes to yield the velocity index,  $V$ .

The above four indices were calculated for both the *TOPEX/Poseidon* as well as the *Geosat* monthly maps. *Geosat* derived monthly maps were decimated to a  $1^\circ \times 1^\circ$  grid and GS indices were calculated similarly except for the position index, which was calculated using the center of the error function used for the *ssh* difference. This

### 3 CALCULATING GULF STREAM INDICES

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was because of differences in the way the mean field was reconstructed in the two data sets as well as differences in spatial resolution. Figures 11 and 12 show the indices for *TOPEX/Poseidon* and *Geosat* for the upstream region ( $73^{\circ}\text{W}$  to  $64^{\circ}\text{W}$ ), while Figures 13 and 14 show the indices for the downstream region ( $63^{\circ}\text{W}$  to  $50^{\circ}\text{W}$ ). Although there is no clear annual cycle apparent in the downstream indices, there is one in the upstream ones. Putting together both time series and fitting an annual signal to them shows a maximum in position (corresponding to a minimum in curvature) in September with a minimum (and a corresponding maximum in curvature) in March. Transport is maximum in October and minimum in April. The amplitudes of the position and transport indices are  $0.21^{\circ}$  and  $0.07\text{m}$  respectively. This implies that the Gulf Stream is stronger and follows a relatively straight and northerly path in fall, and then shifts to a southerly position with weaker transport accompanied by more meandering in spring. A detailed analysis of the index cycle may be found in [7].

### Acknowledgements

Support for this project was provided by NASA under contracts NAGW-1666 and NAGW-4806.

### 3 CALCULATING GULF STREAM INDICES

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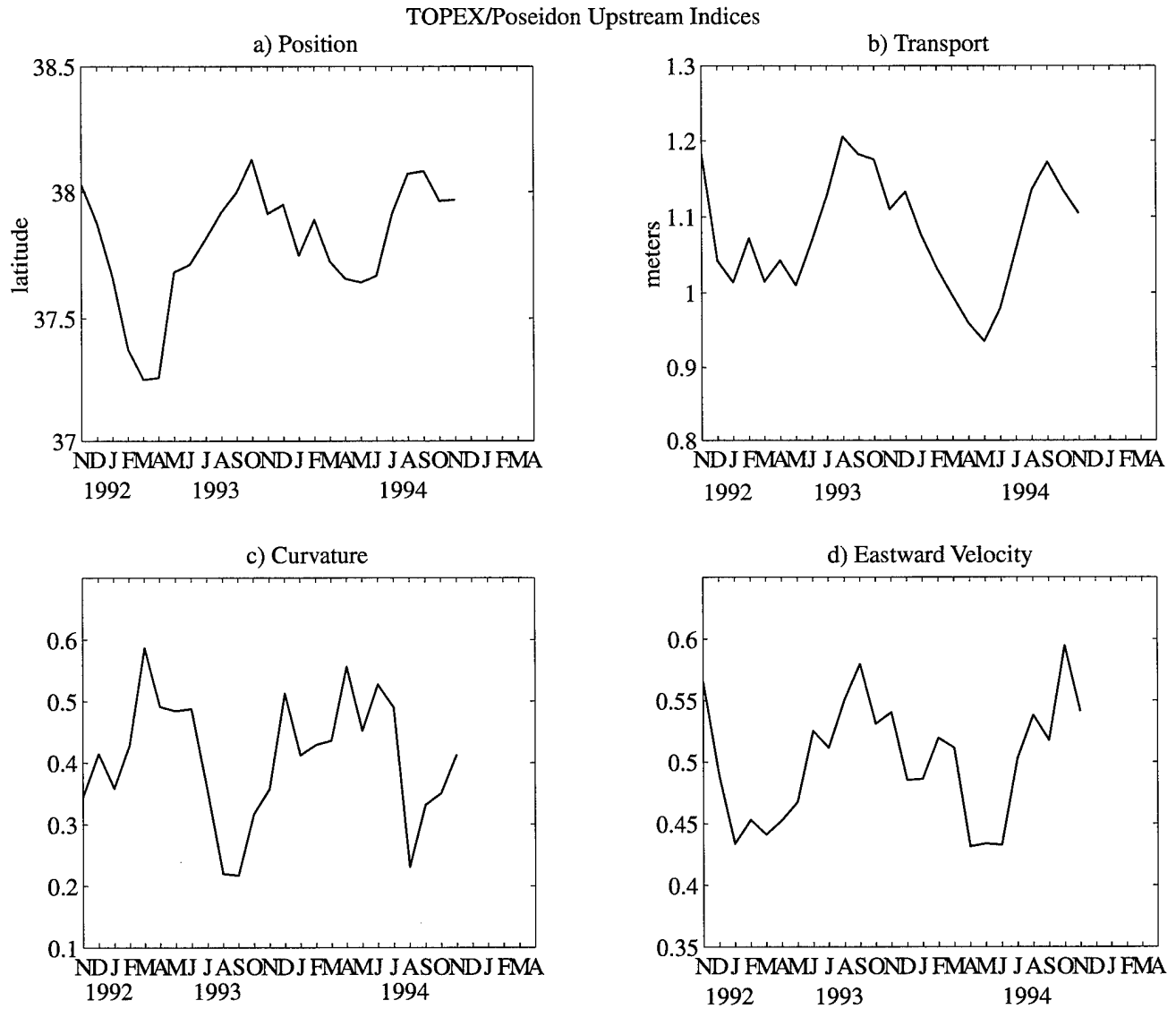


Figure 11: Zonal indices for 73°W to 64°W from *TOPEX/Poseidon*.



### 3 CALCULATING GULF STREAM INDICES

Geosat Upstream Indices

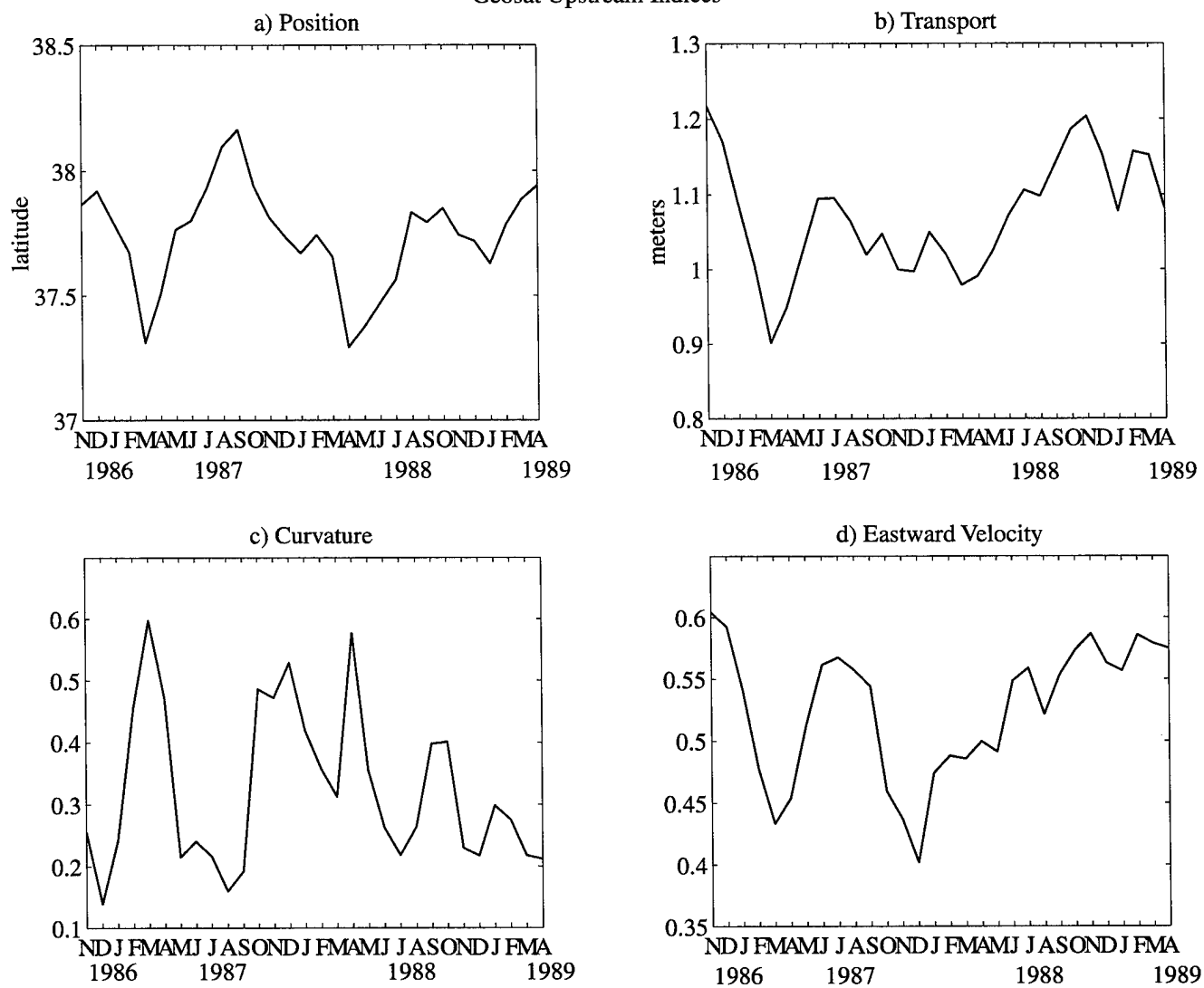


Figure 12: Zonal indices for 73°W to 64°W from *Geosat*.

### 3 CALCULATING GULF STREAM INDICES

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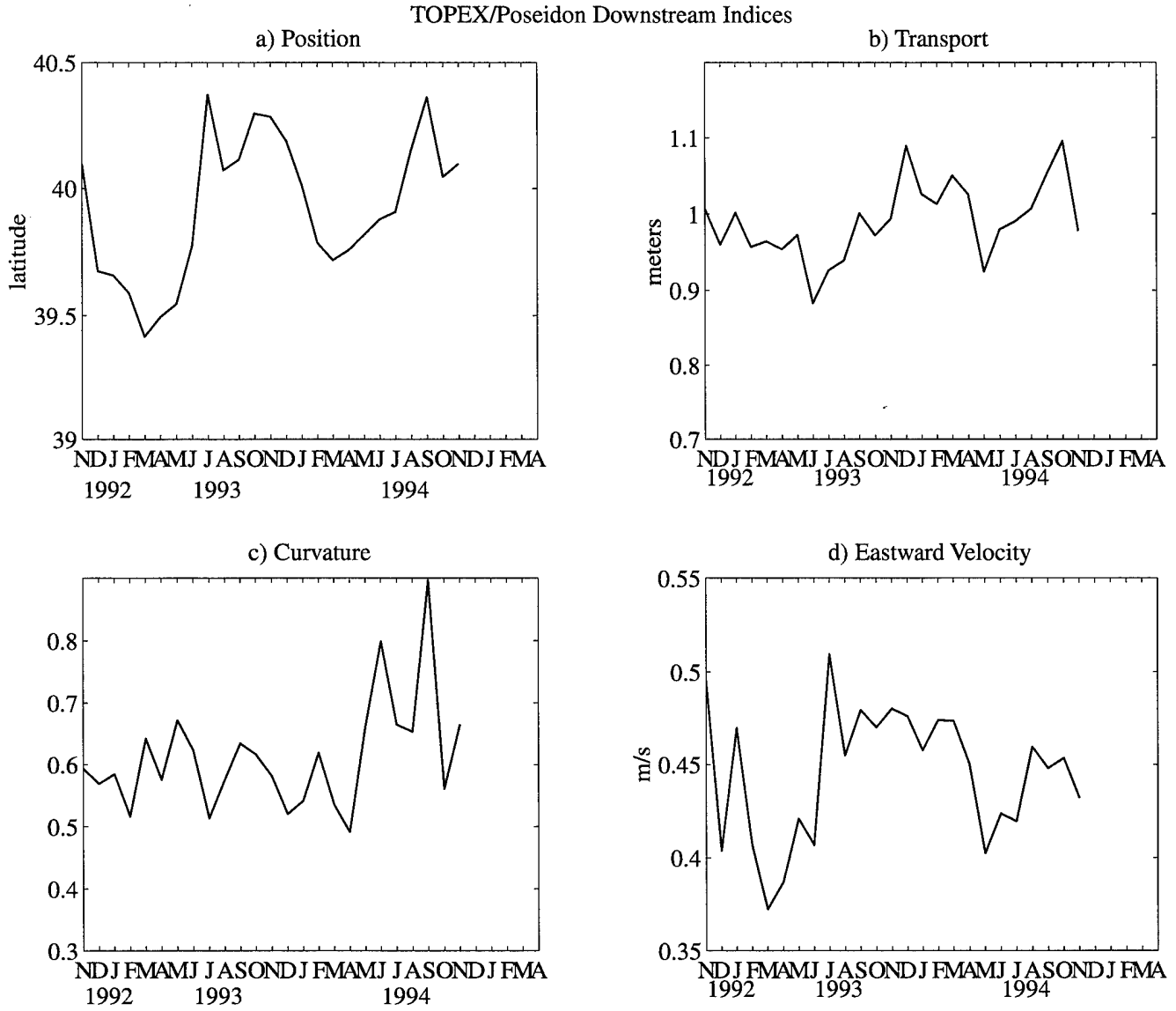


Figure 13: Zonal indices for 63°W to 50°W from *TOPEX/Poseidon*.

### 3 CALCULATING GULF STREAM INDICES

#### Geosat Downstream Indices

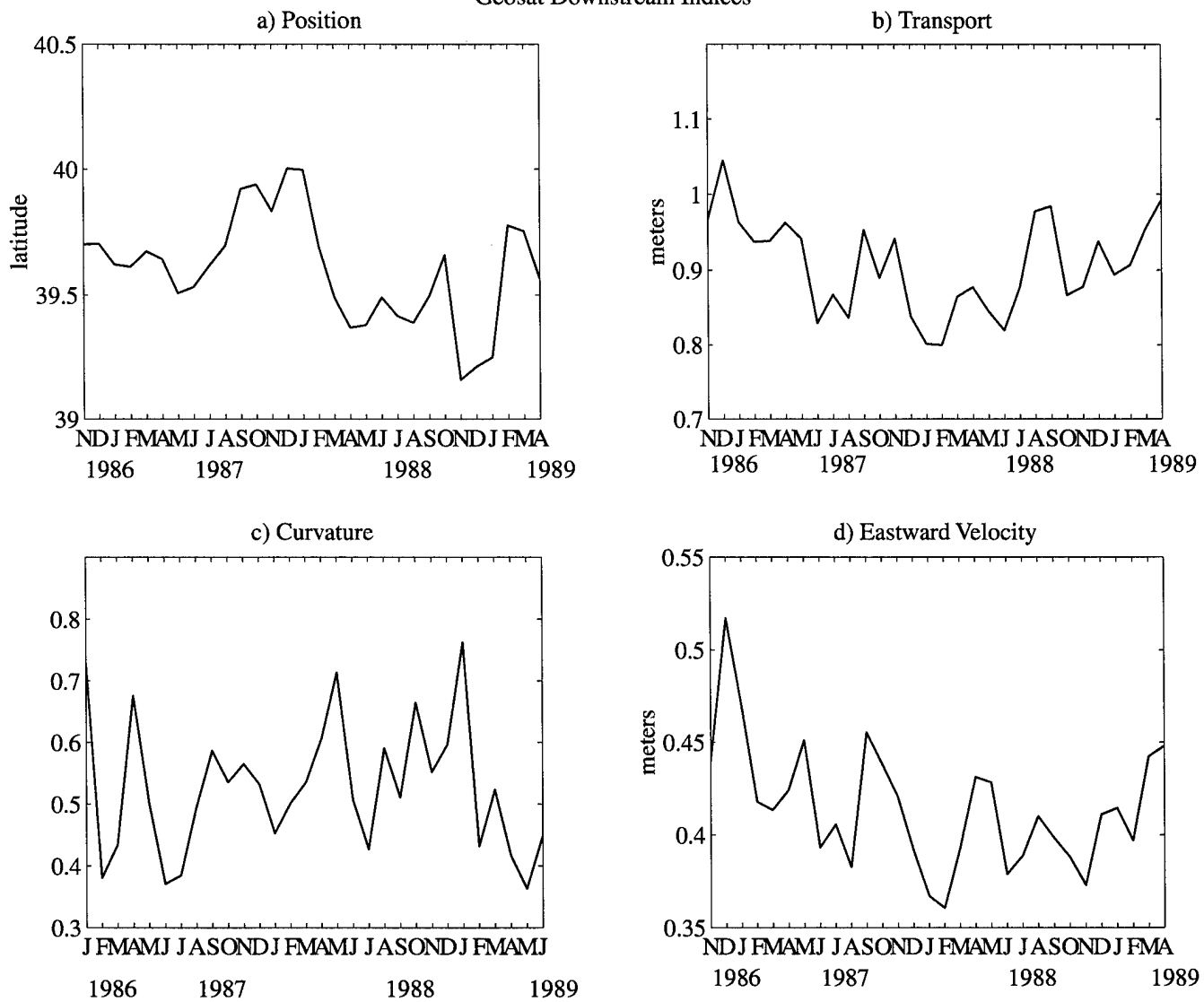


Figure 14: Zonal indices for 63°W to 50°W from *Geosat*.

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- [6] Qiu, B., 1994. Determining the mean Gulf Stream and its recirculations through combining hydrographic and altimetric data, *J. Geophys. Res.*, **99**, 951–962.
- [7] Kelly, K. A., S. Singh and R. X. Huang. The index cycle in the Gulf Stream. *in preparation*

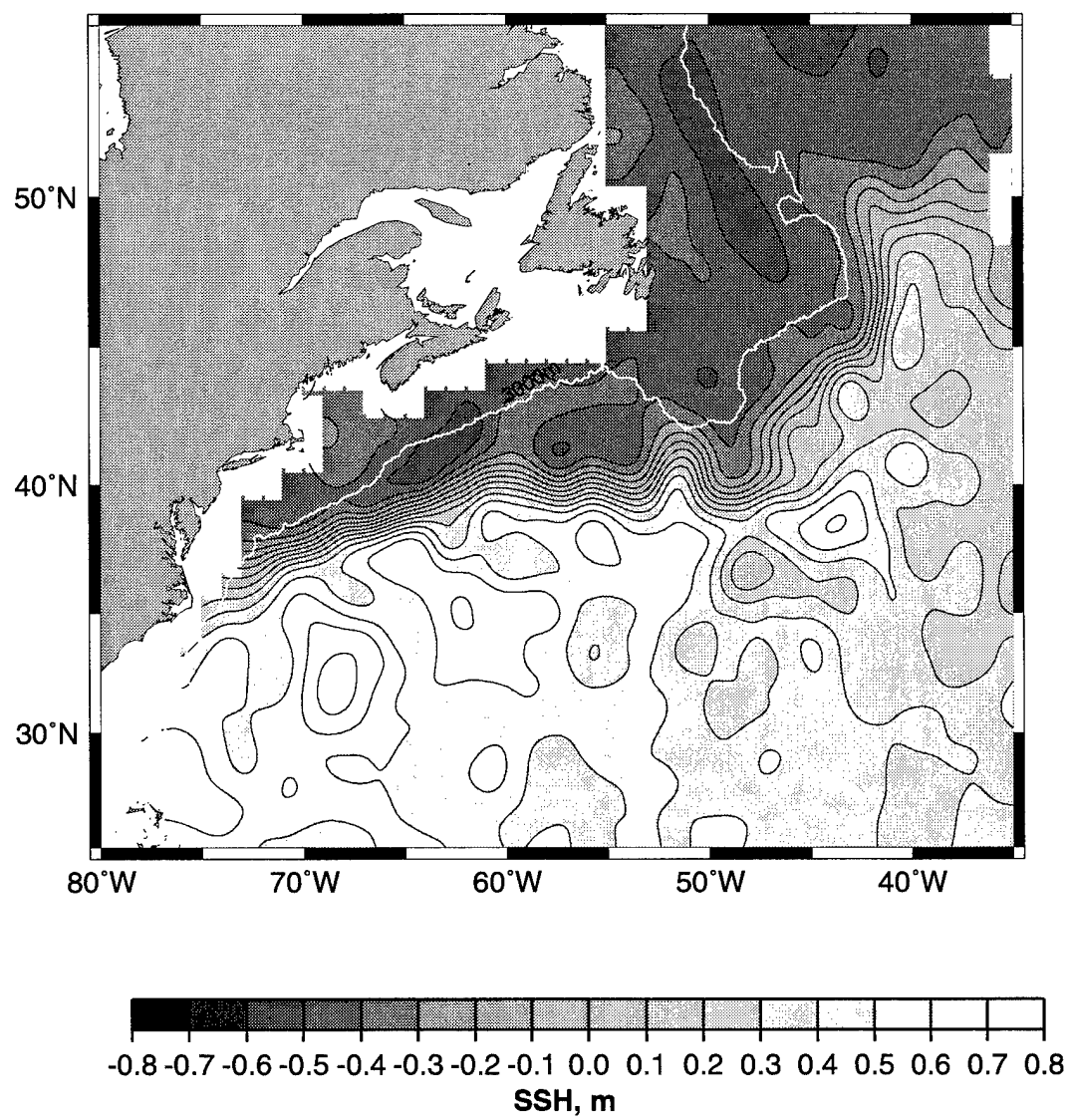
## Appendix

Monthly maps of total *ssh* derived from *TOPEX/Poseidon* altimeter data are included for the period from November 1992 to November 1994. The 3000 meter isobath (white contour line) is included in the maps for reference.

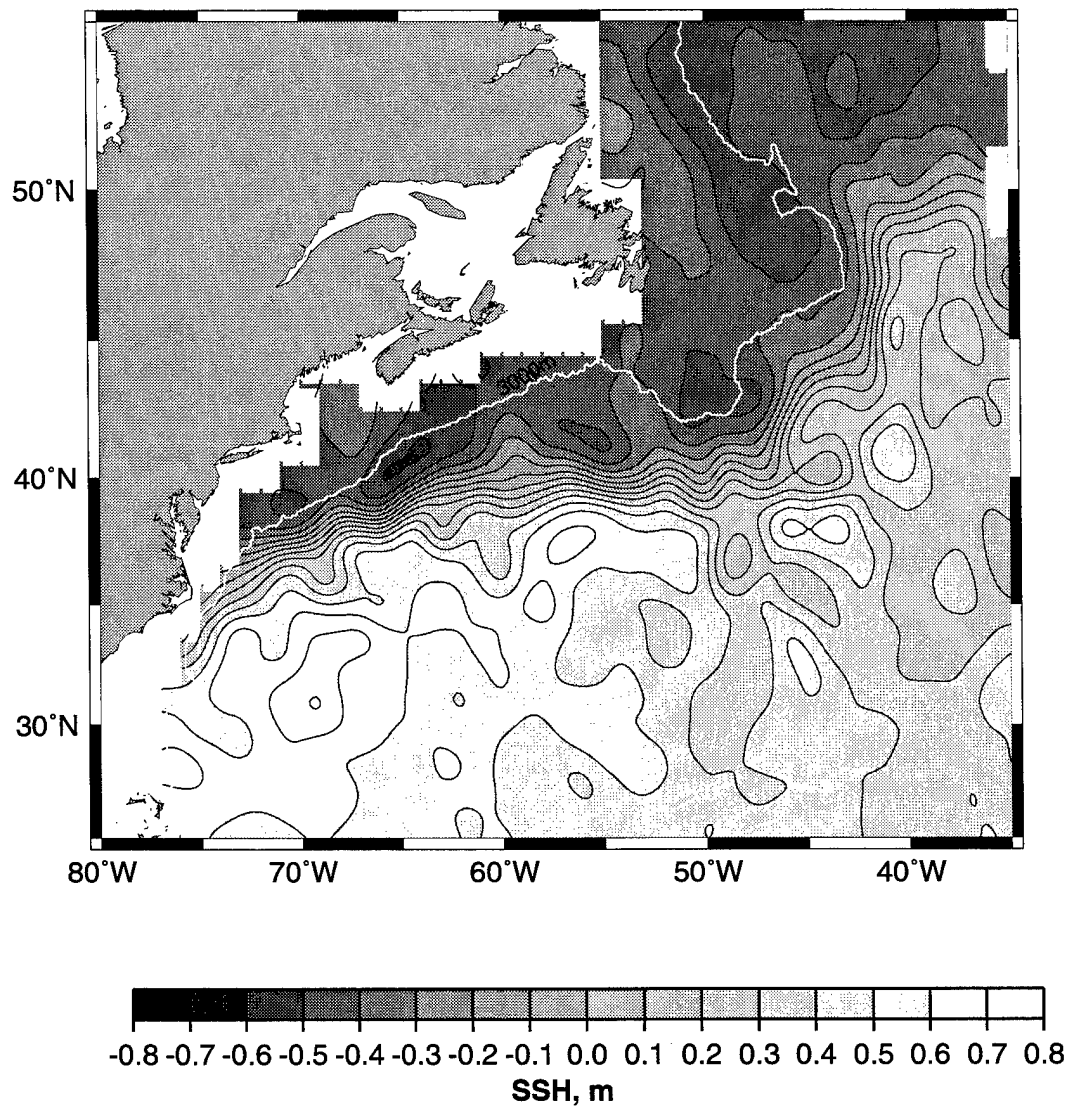
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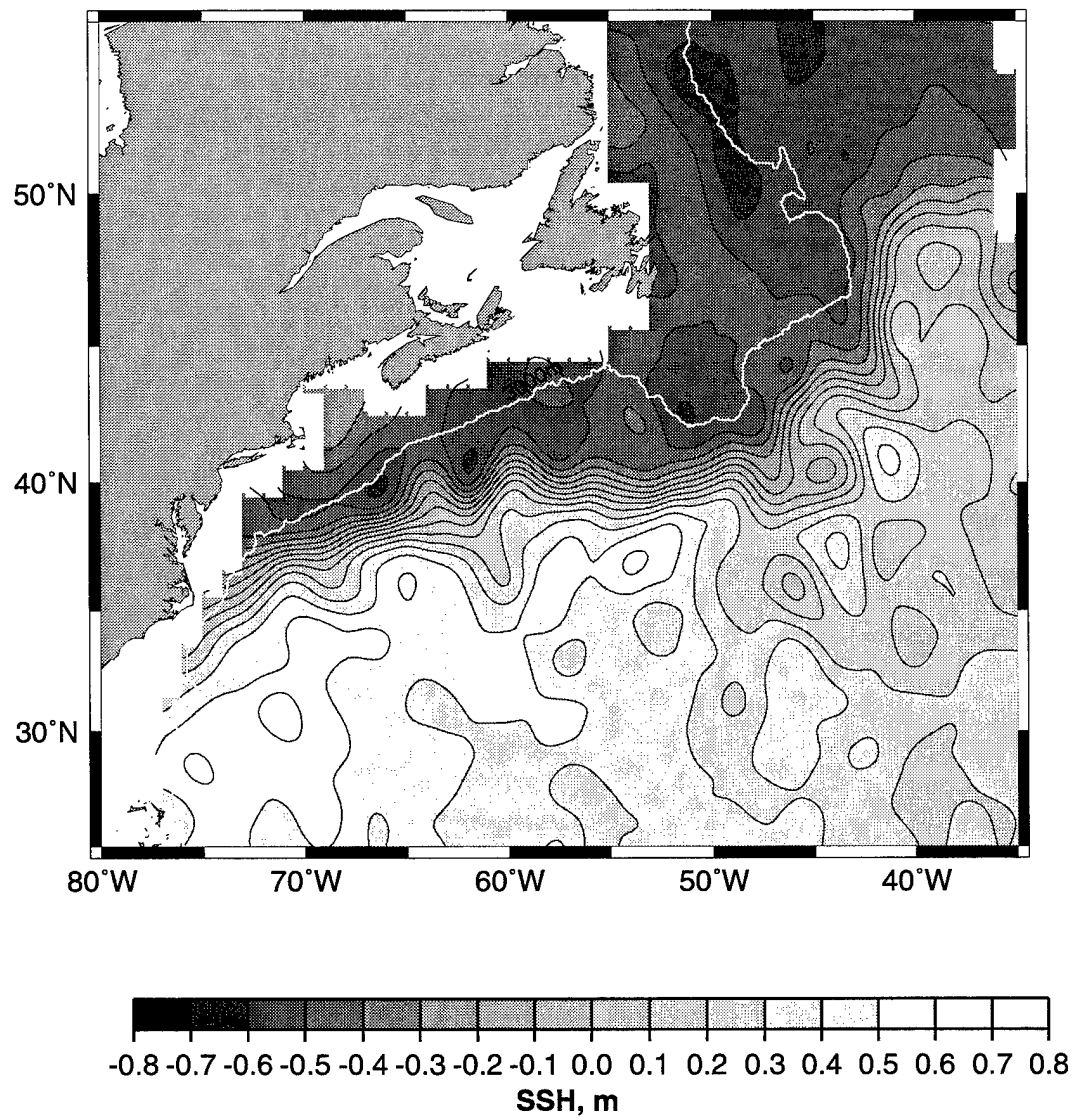
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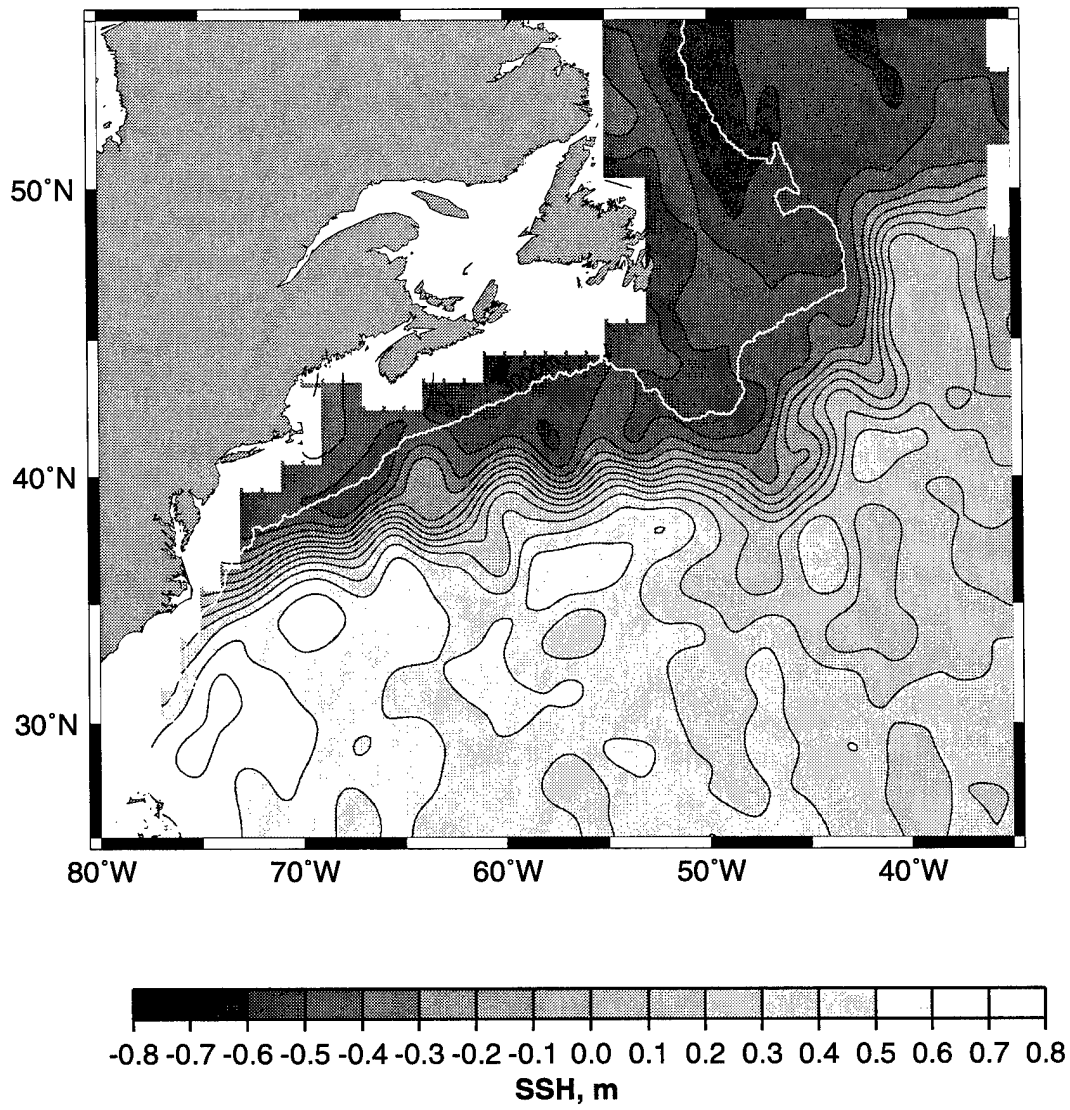
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January 93





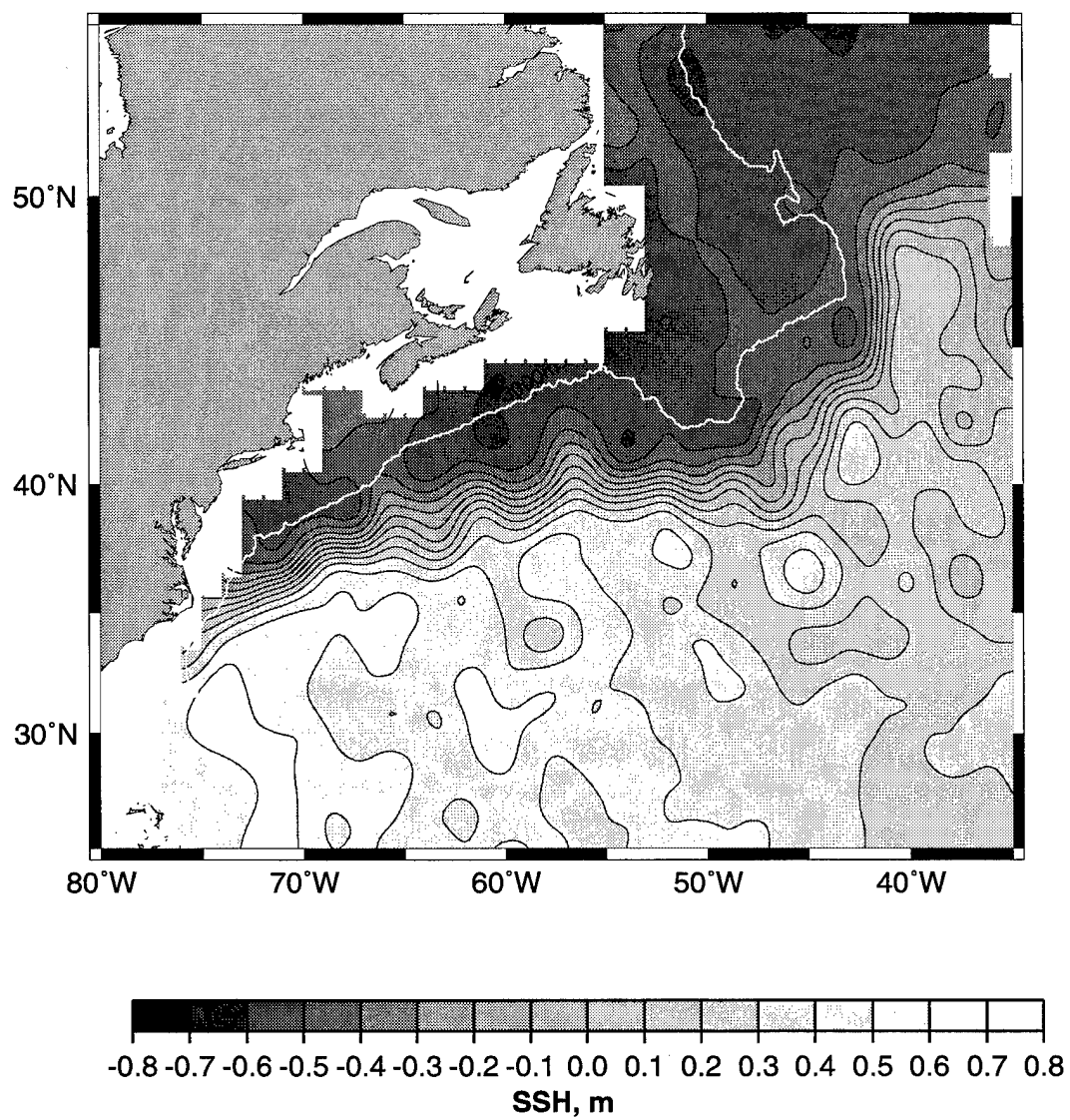
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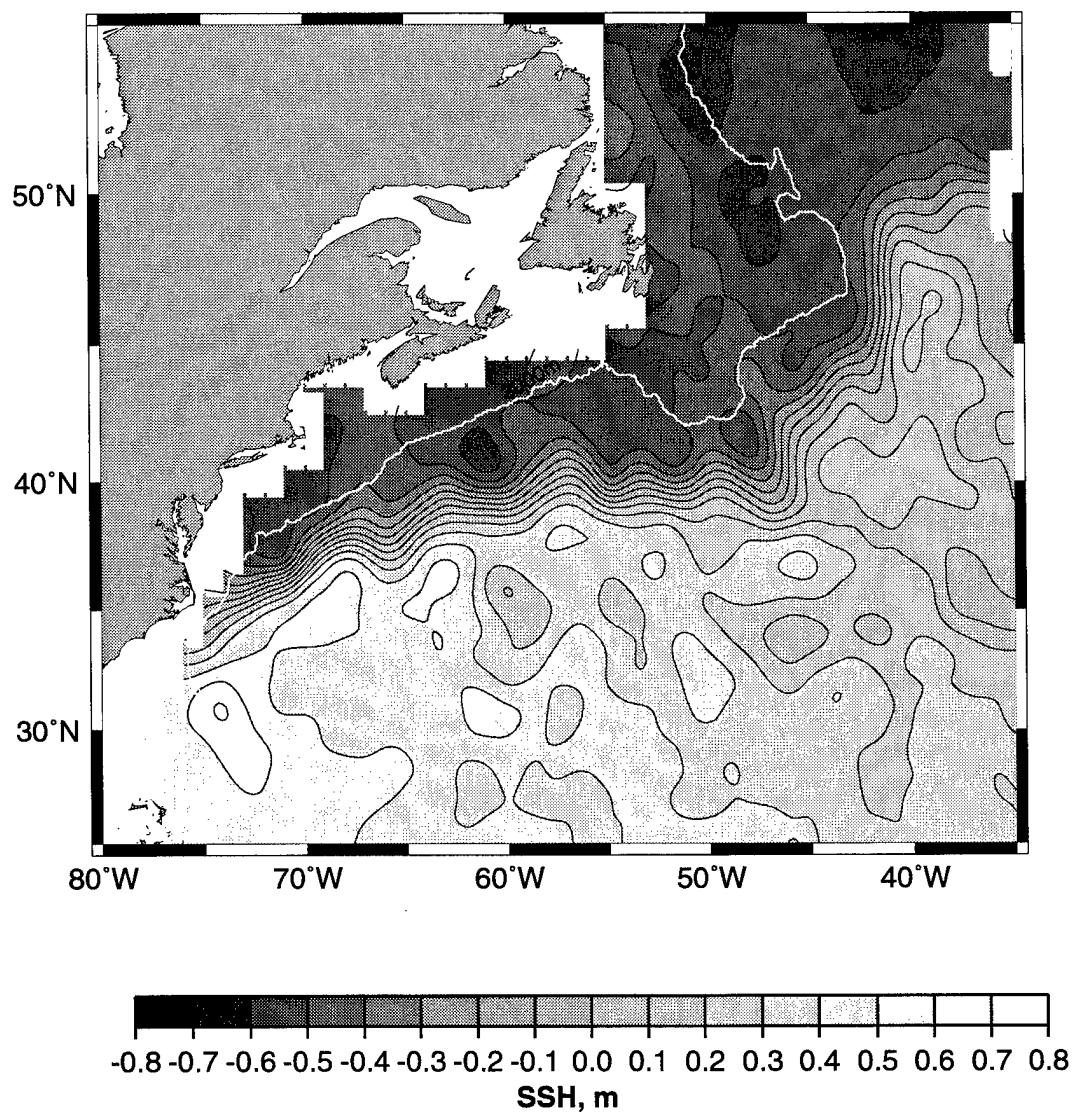
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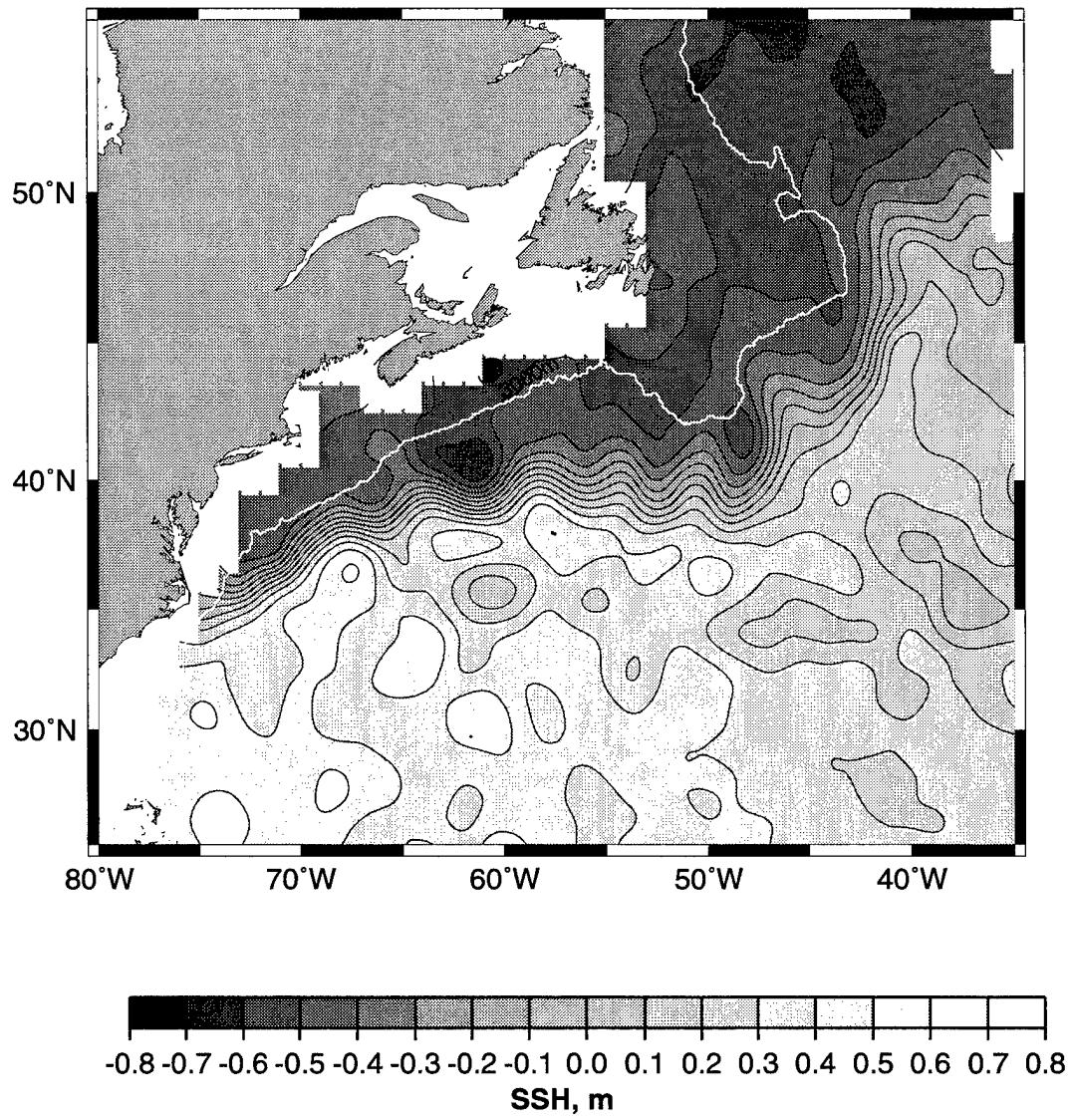
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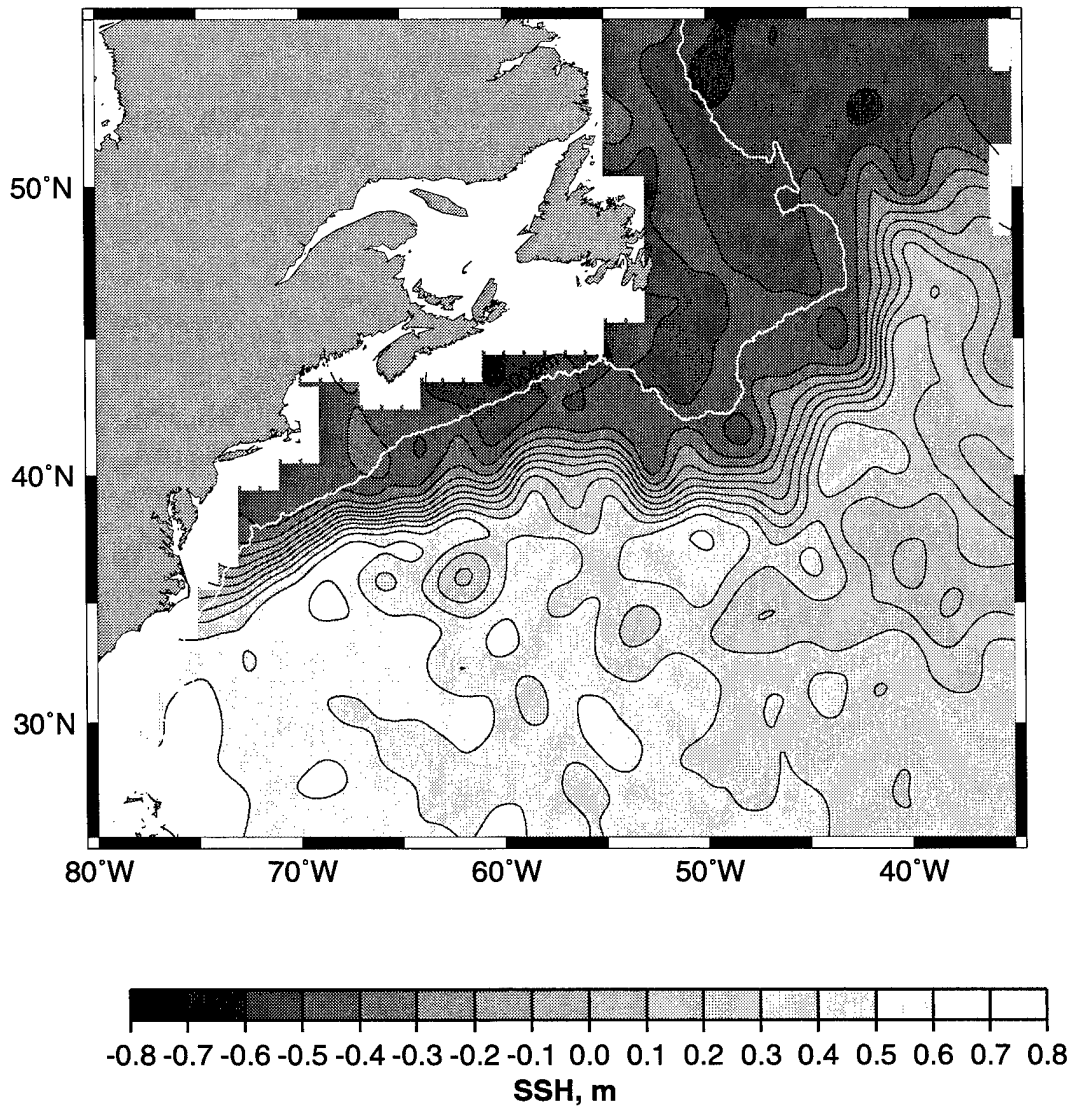
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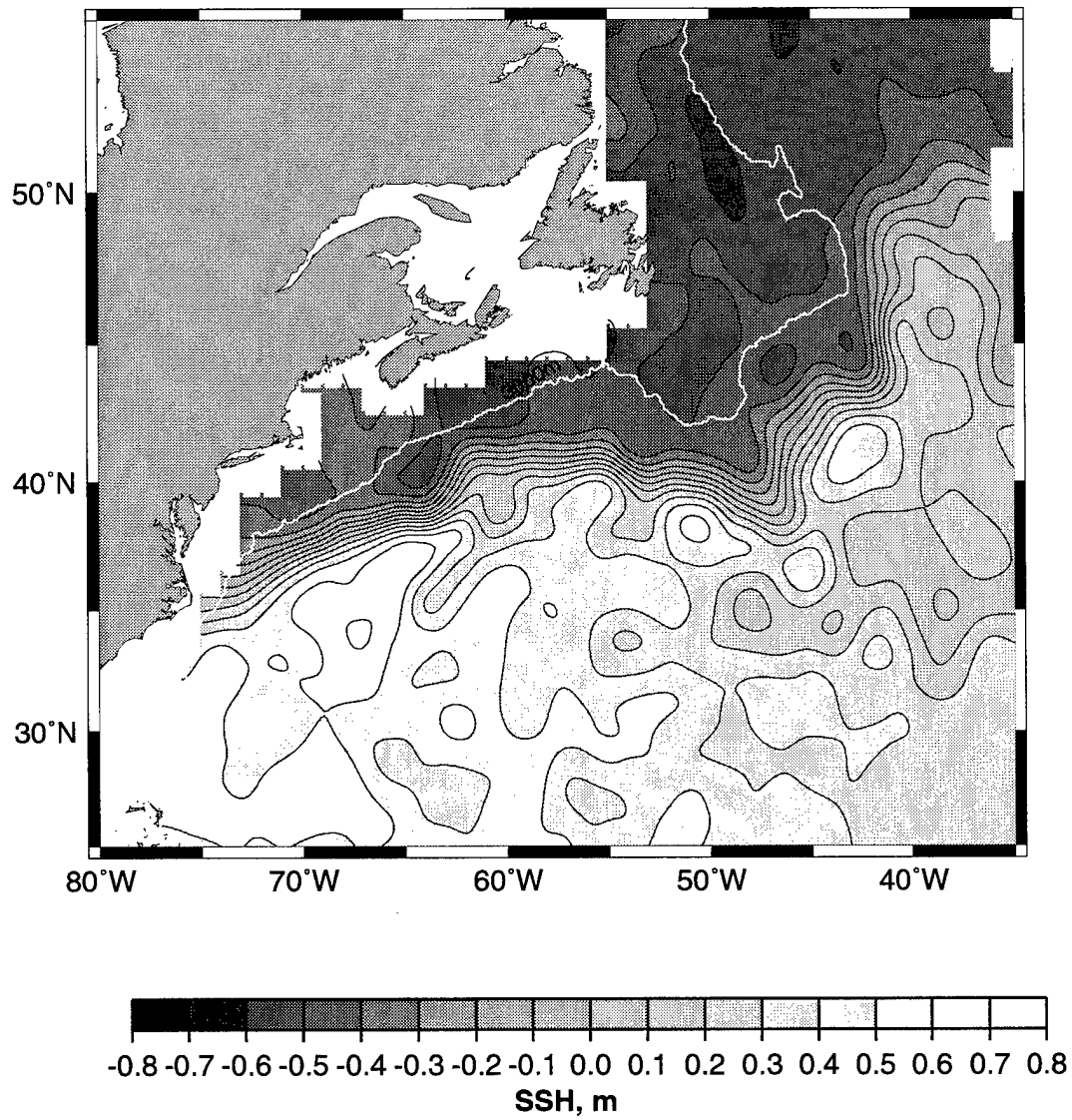
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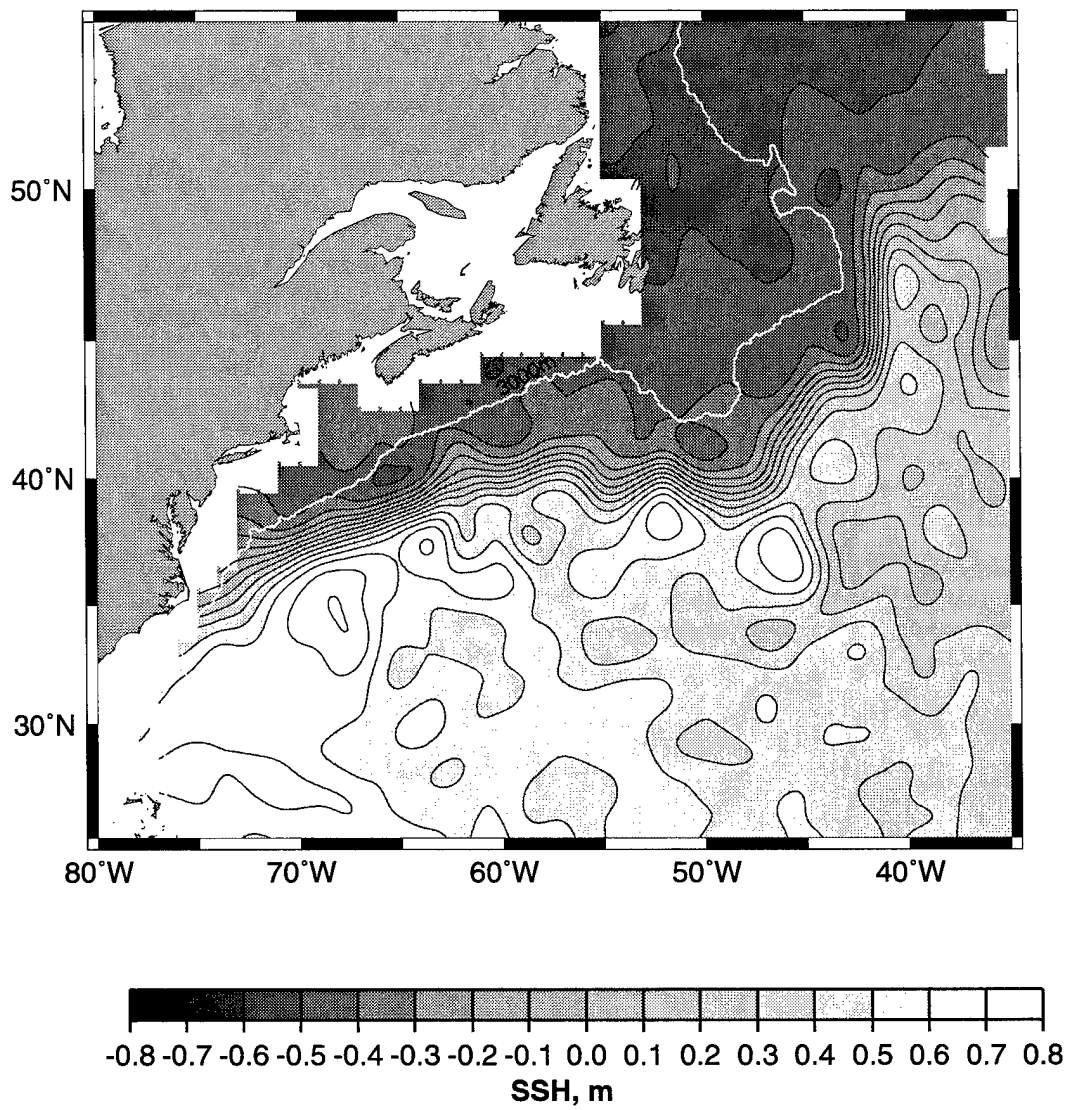
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July 93



August 93

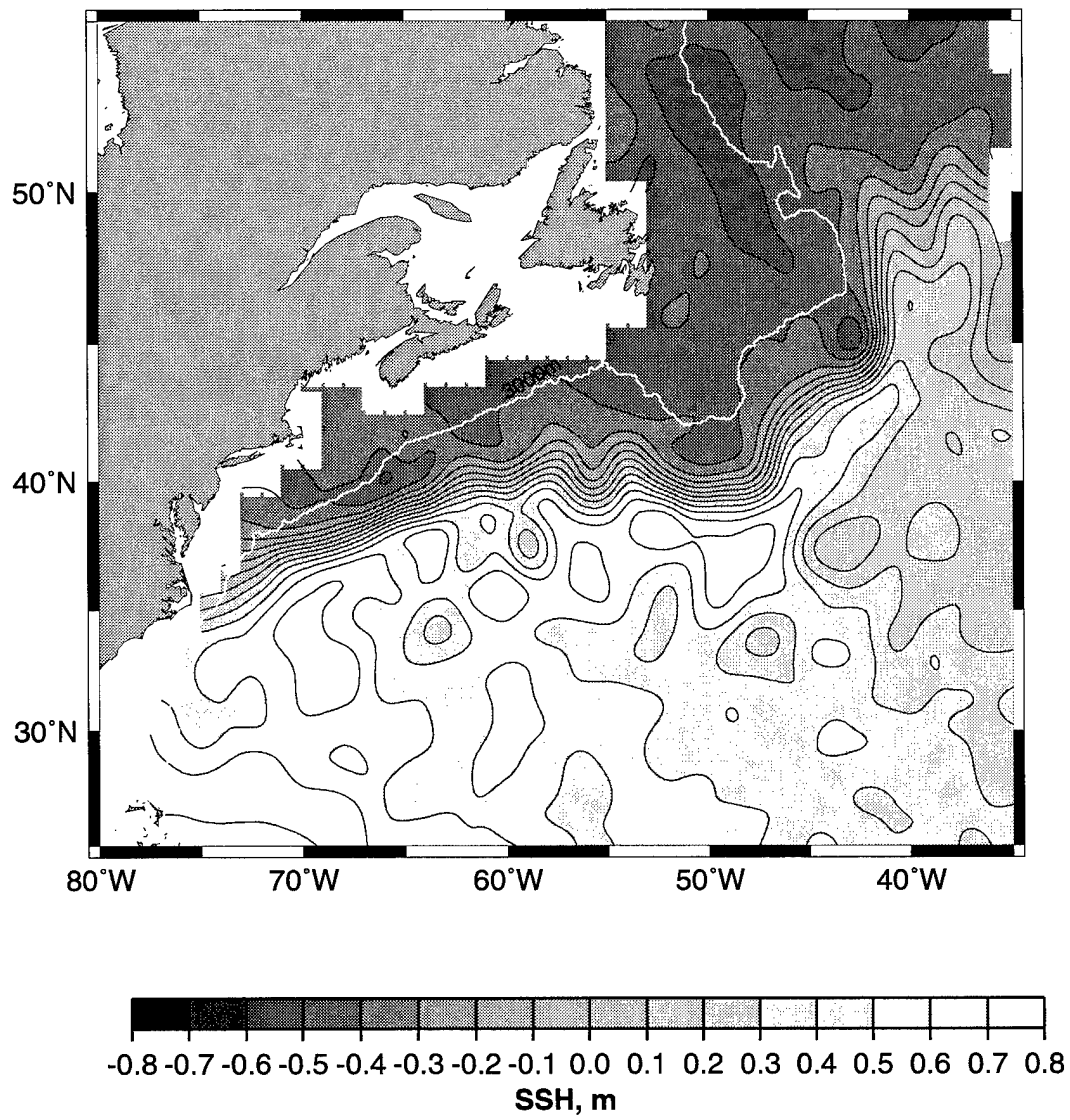




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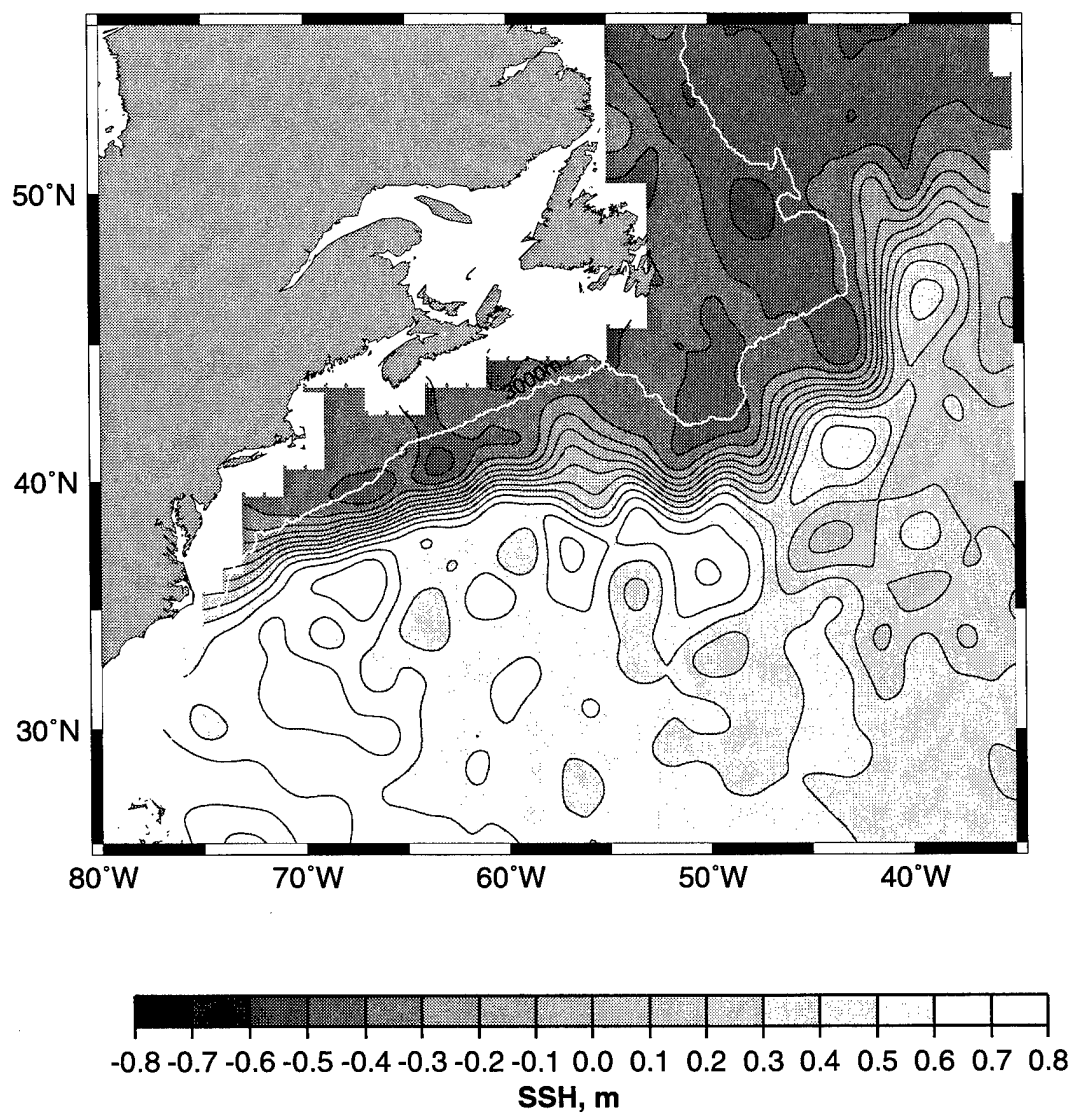
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September 93





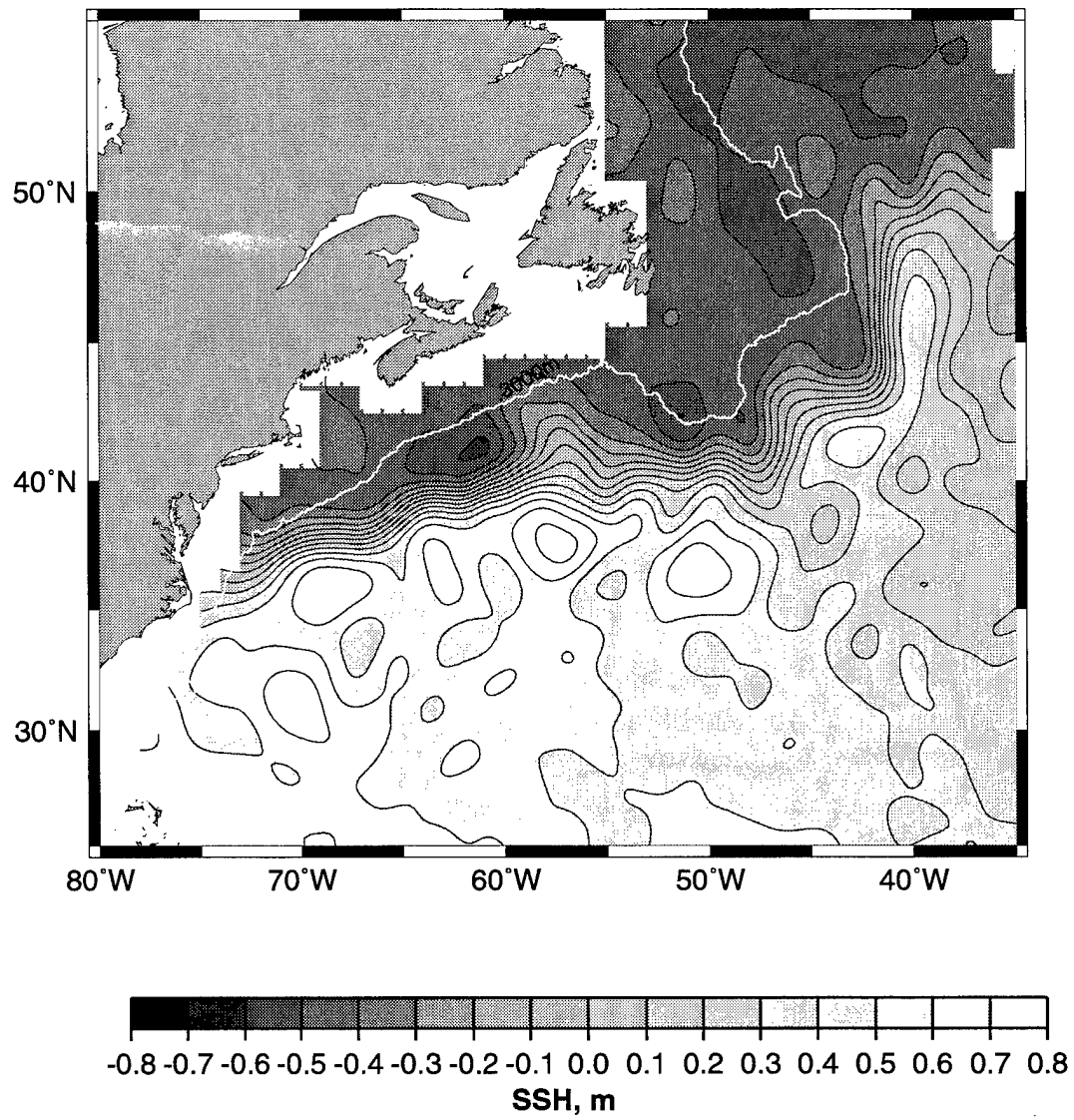
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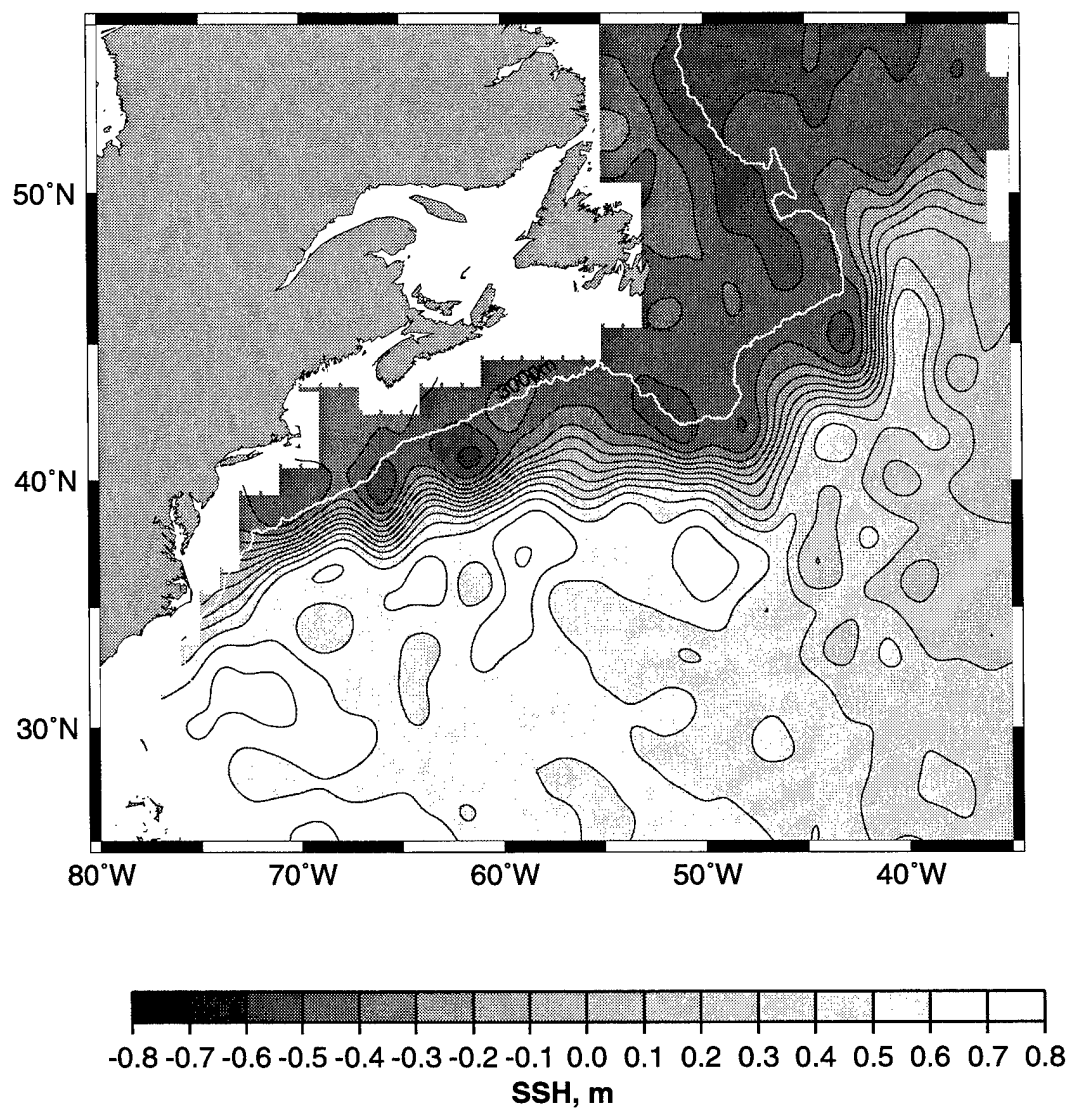
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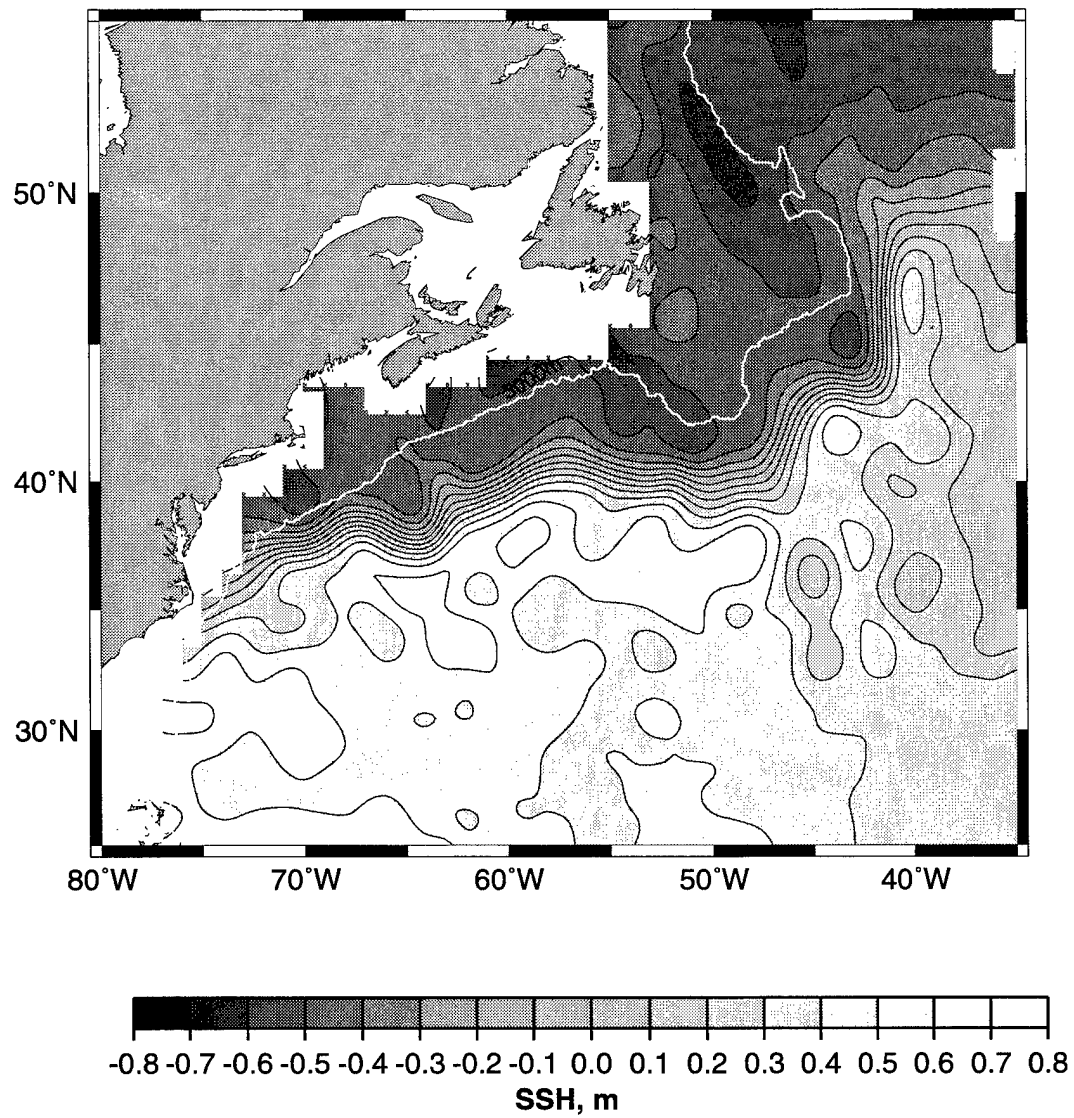
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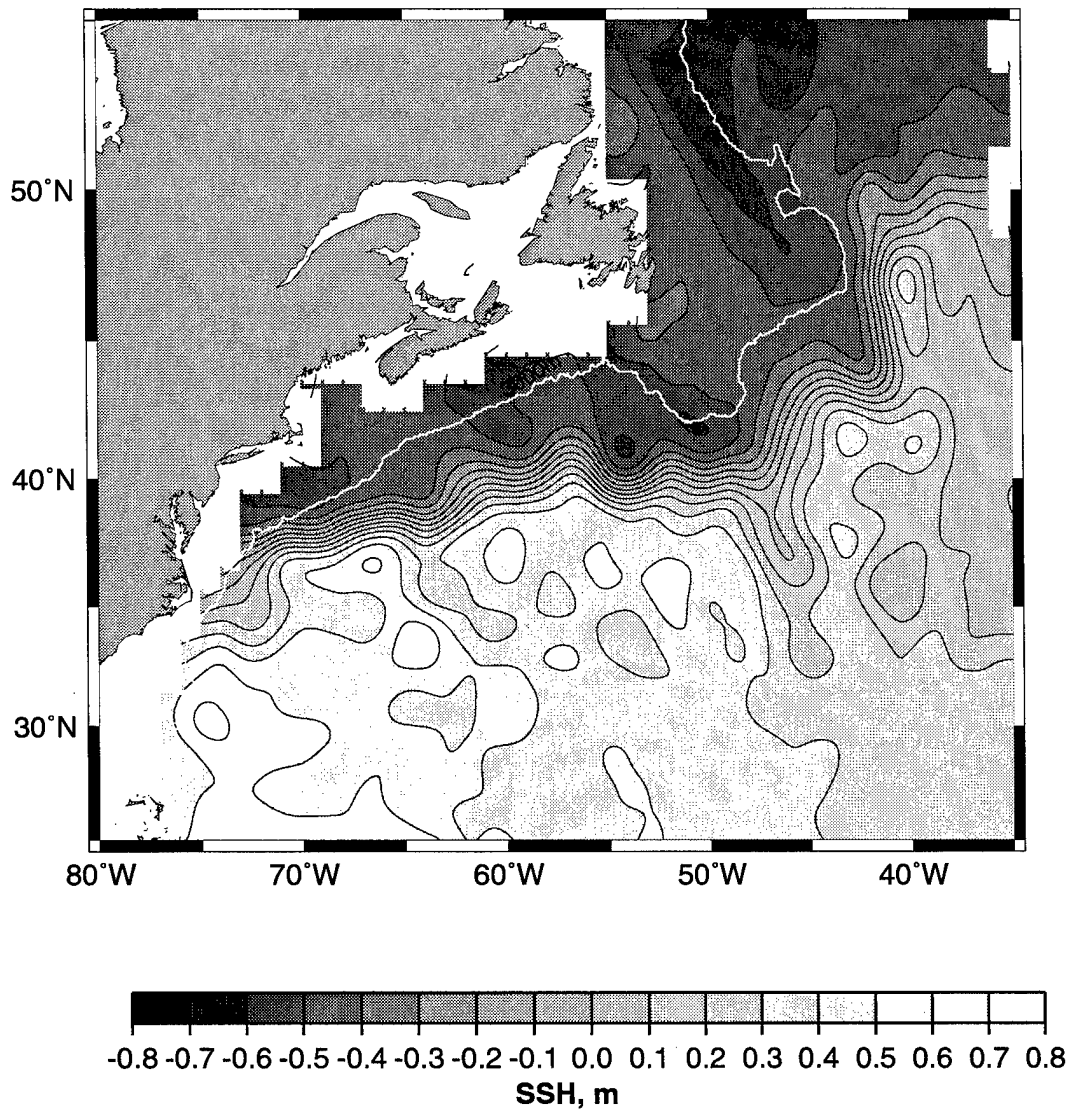
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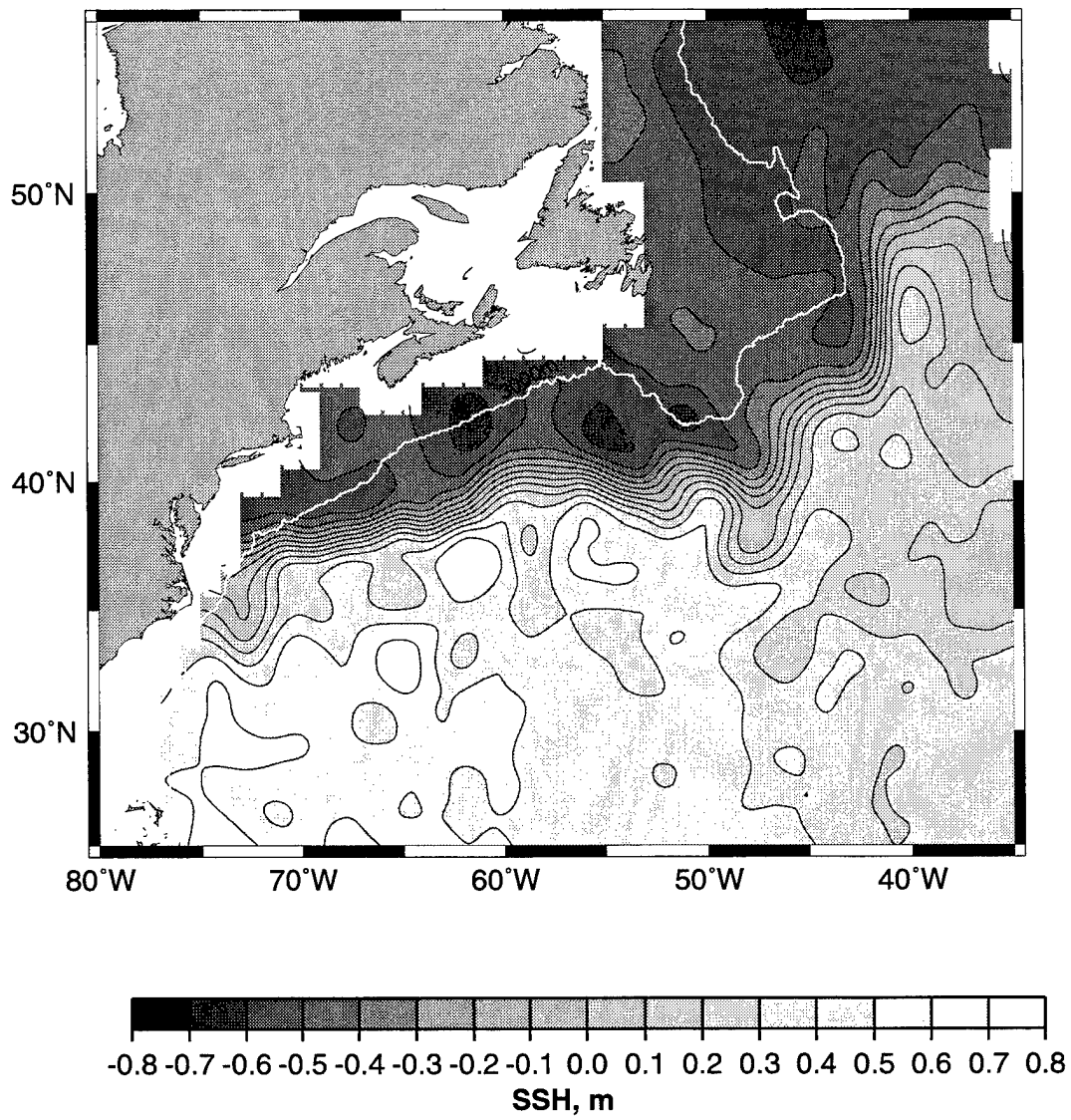
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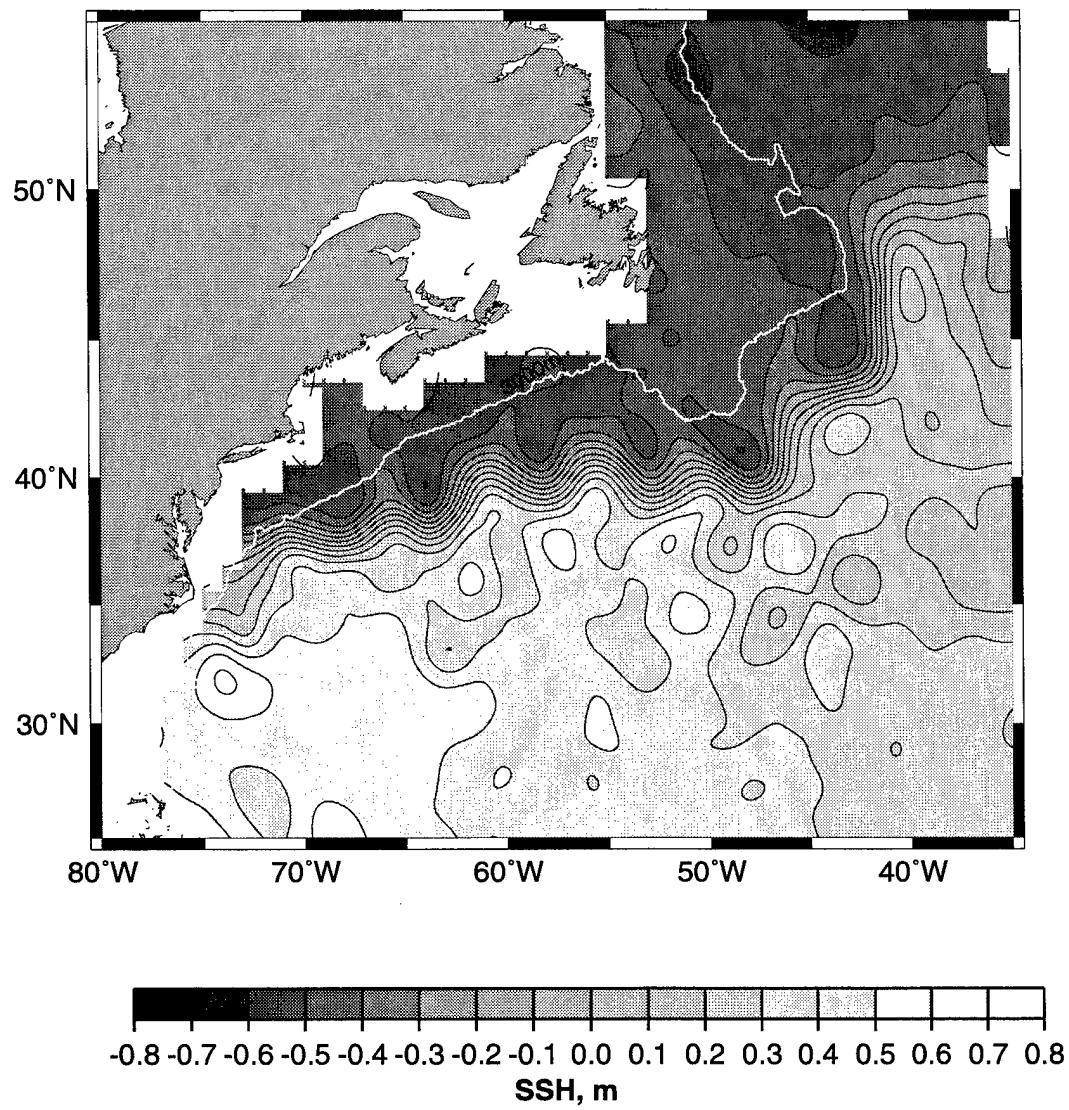
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April 94

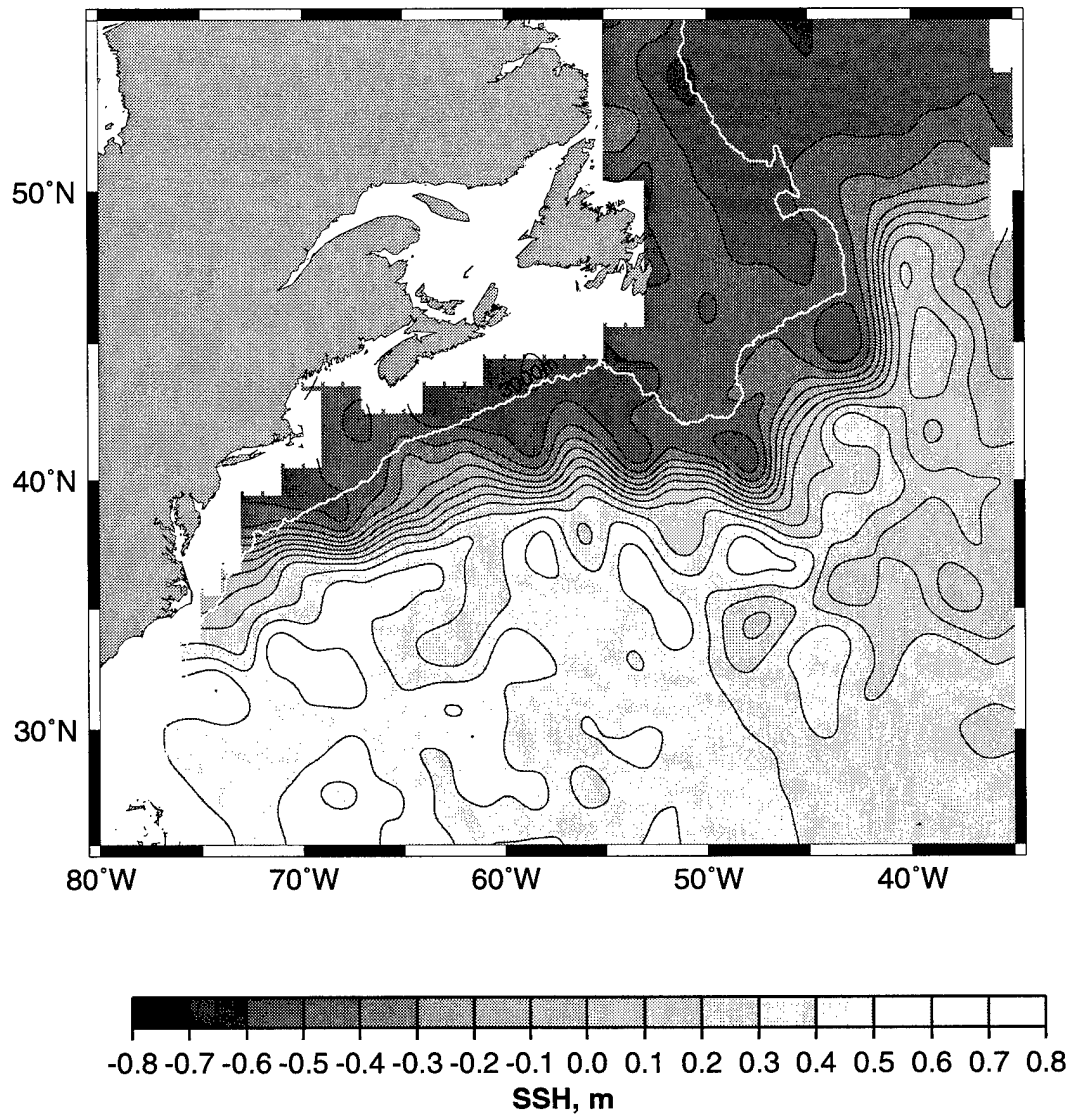




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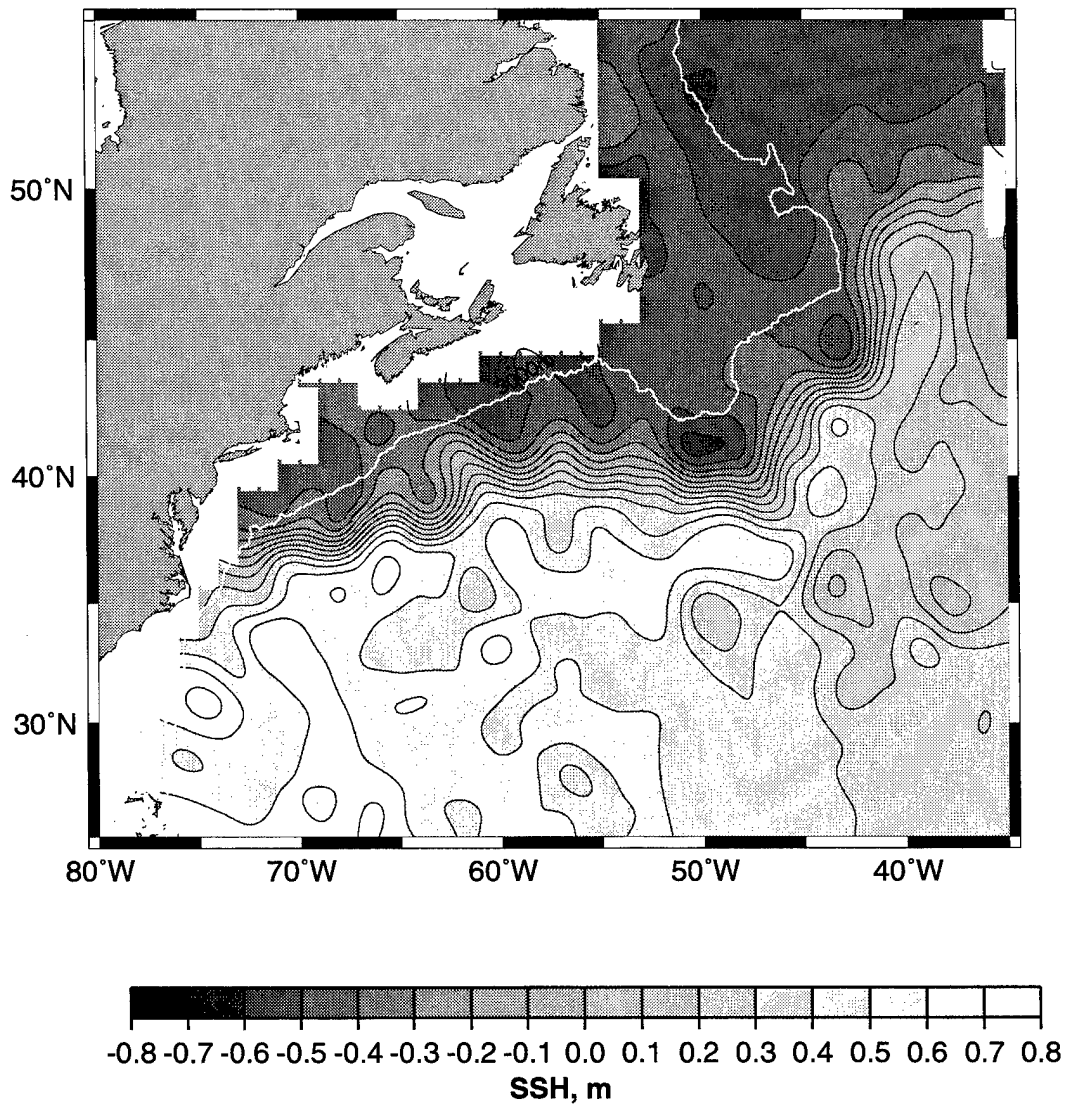
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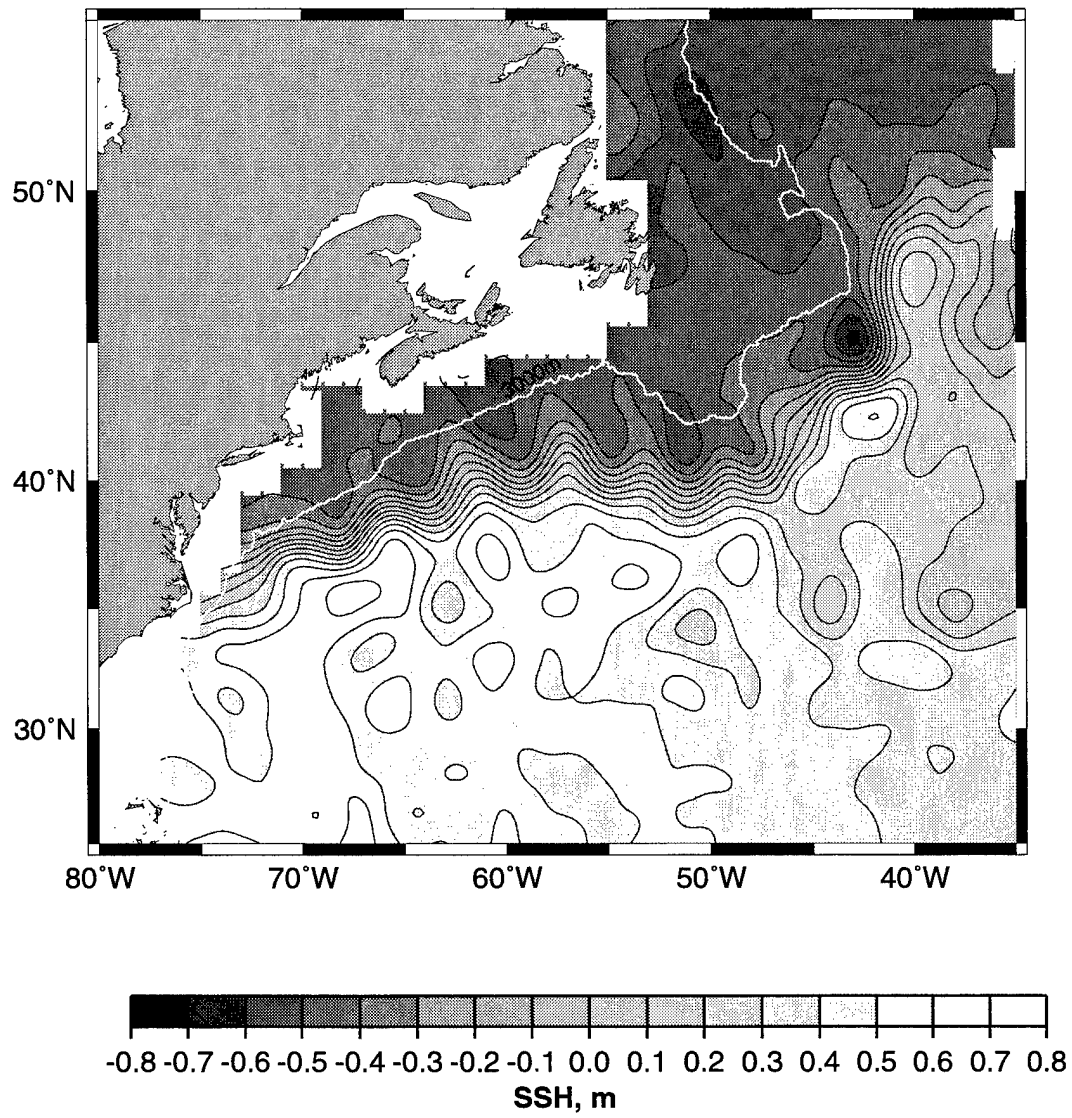
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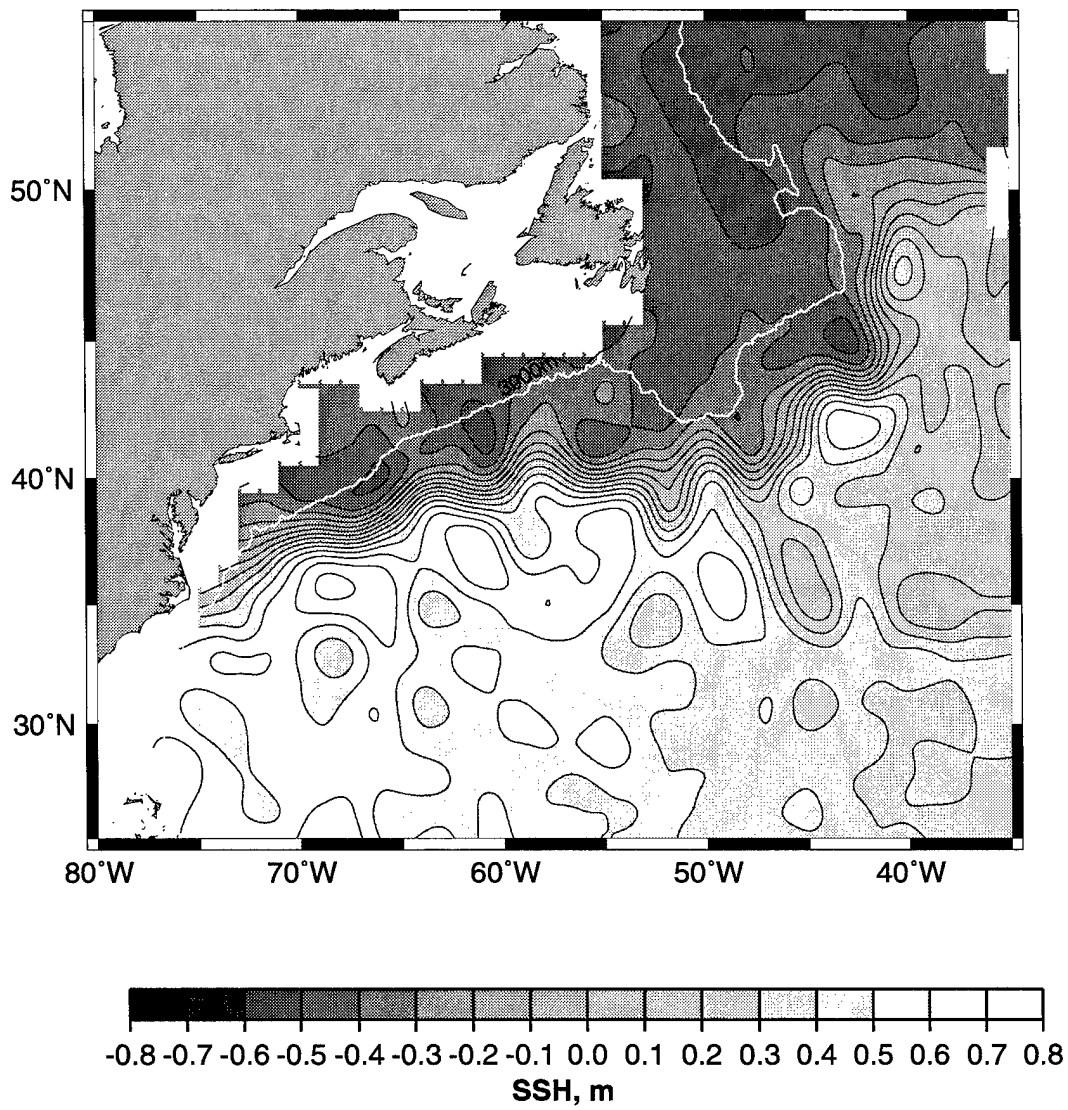
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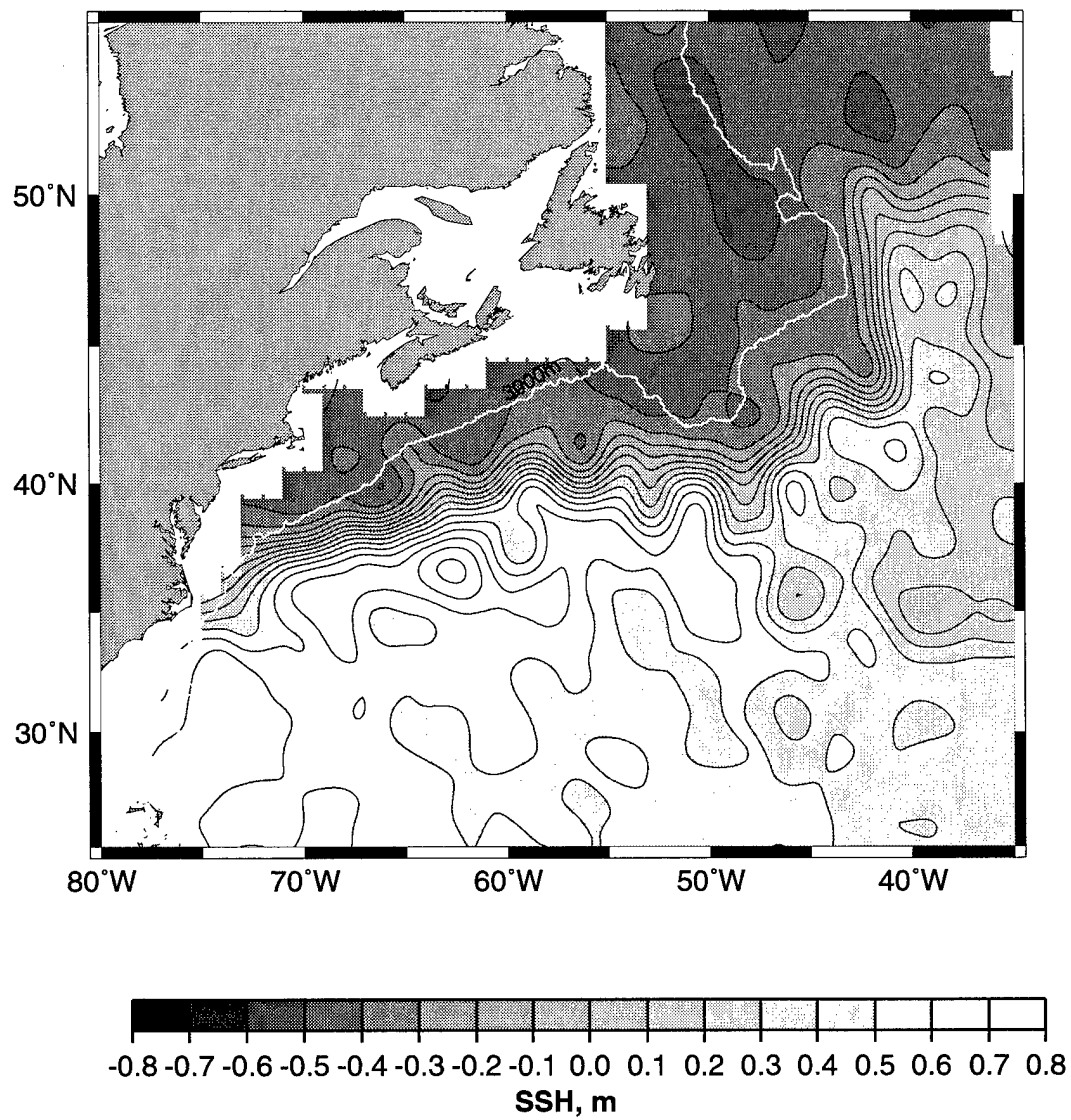
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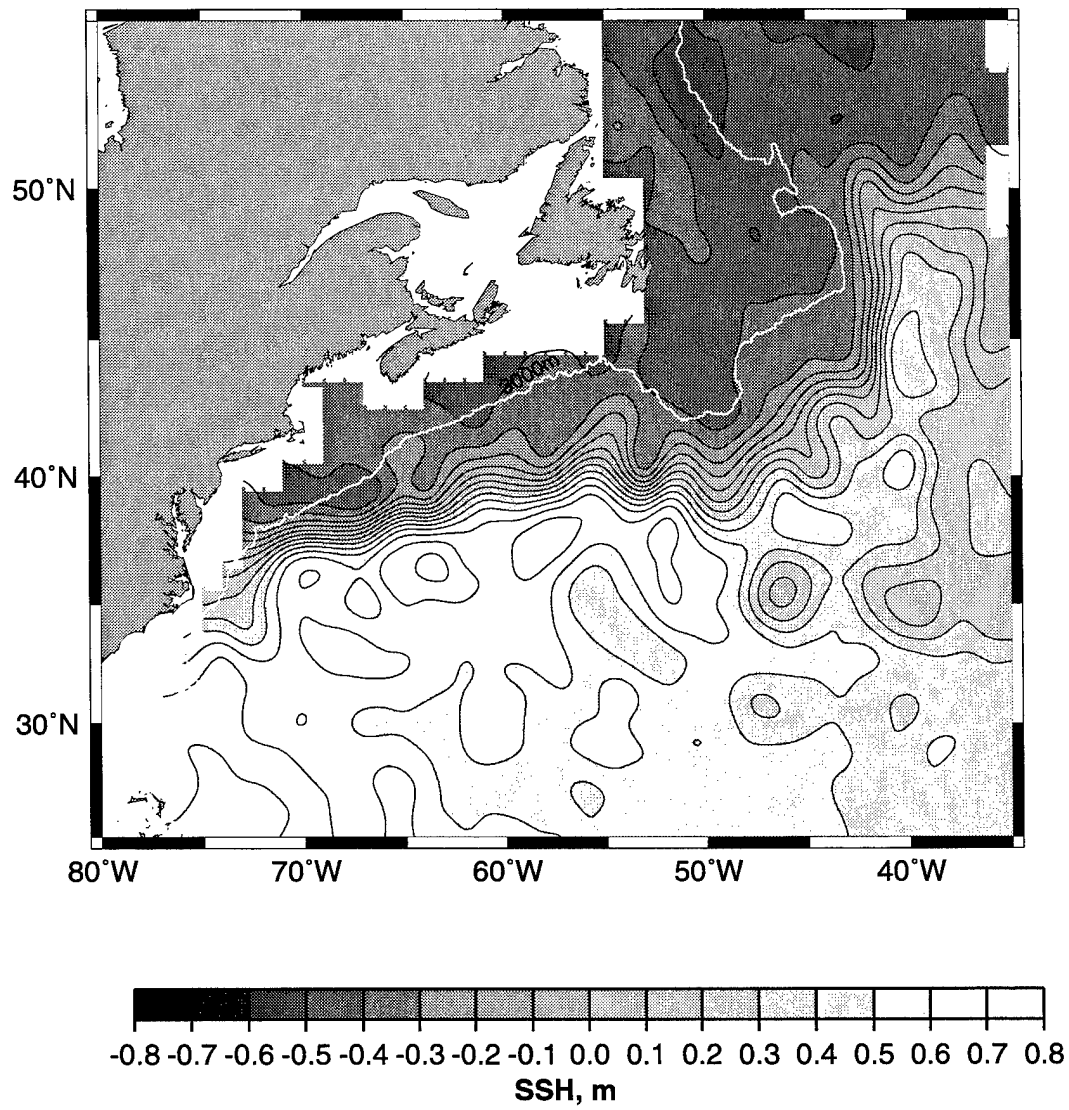
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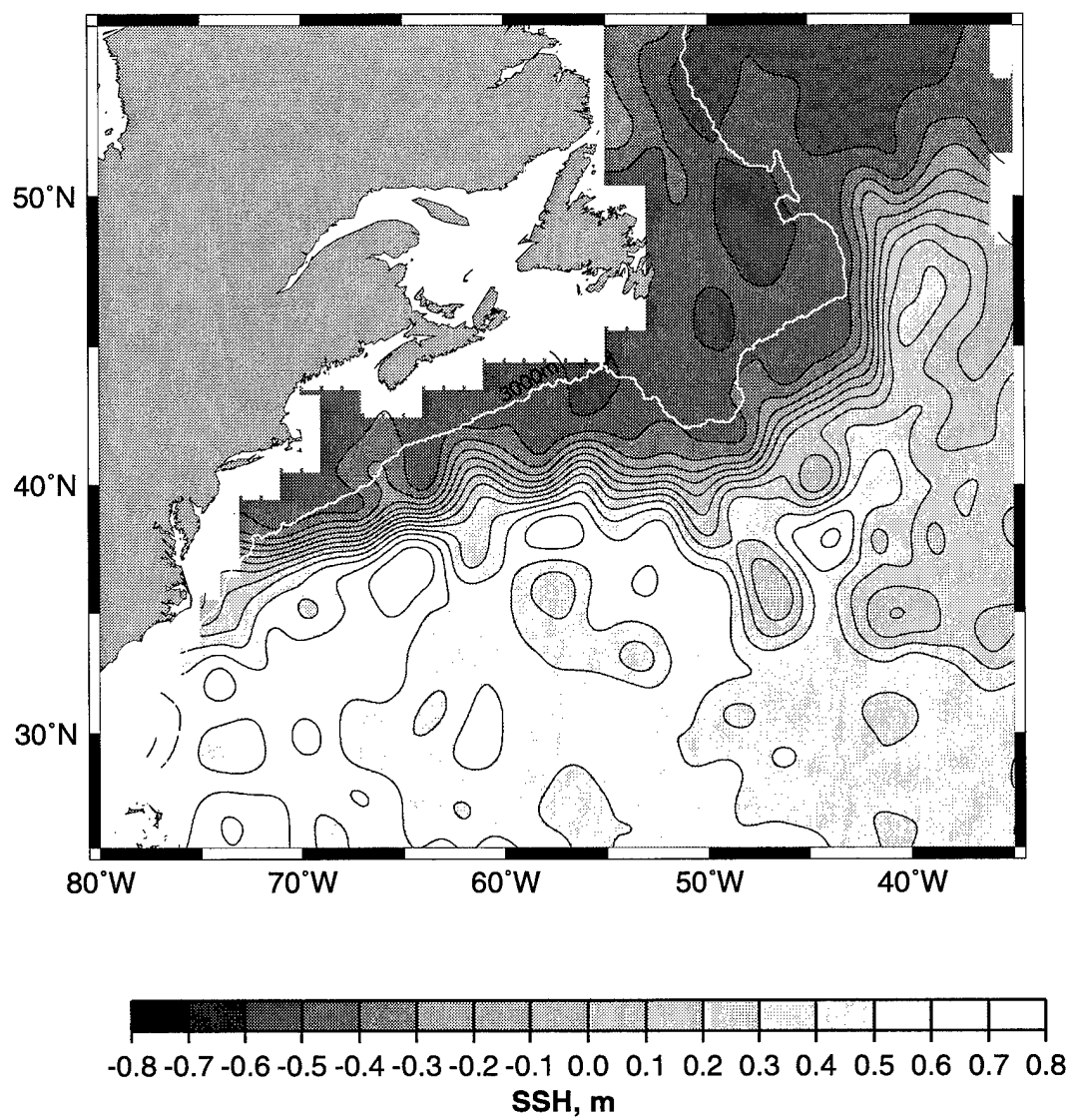
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